TEXAS WATER DEVELOPMENT BOARD

REPORT 128

SIMULATION OF WATER QUALITY

IN STREAMS AND CANALS

Theory and Description of the
QUAL-I Mathematical Modeling System

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PREFACE

The field survey data used for verification of the dissolved oxygen option of the QUAL-I simulation model were preliminary in nature at the time of preparation of the report manuscript. Additional field data have subsequently become available and further simulation analyses have been performed.

These new data indicate that several existing run-of-the-river impoundments were not included in the initial model verification studies documented herein. These impoundments caused the measured dissolved oxygen concentrations to be lower than those predicted by the model.

The analyses performed subsequent to the preparation of this report have shown that a considerably more detailed prototype representation is obtainable than that shown in Figure 11 of this report. Simulation with the impoundments included as model input resulted in predicted dissolved oxygen concentrations which were essentially in exact agreement with all of the measured values shown herein. The increased accuracy of simulation was due solely to more accurate representation of the physical system, and no changes to the computer code presented in this report were necessary. This finding emphasizes the requirement that the model input data should be as complete as possible if accurate simulation is desired.

Arden O. Weiss, Director
Systems Engineering Division
January 13, 1971

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ABSTRACT

Generalized and complex numerical quality routing models have been available that are capable of describing the waste transport and assimilation phenomena of the receiving waters throughout a river basin system. However, simpler but more restrictive models can be applied and answers more rapidly obtained in certain situations involving specific wastes in well defined, relatively uncomplicated open-channel water transfer systems (streams and pump canals). Such models are very useful for studying small segments of a river basin, and can be designed to provide fast and economical solutions to given problems.

The set of interrelated quality routing models (QUAL-I) described in this report is useful for the prediction of the temporal and spatial distribution of temperature, biochemical oxygen demand and dissolved oxygen, and conservative minerals within a segment of a river basin. The governing differential equation is solved by an implicit-finite-difference technique under the assumption that advection along the primary axis of flow (longitudinal axis of the stream channel) is the primary mode of transport.

Comparison of the predictions of this modeling system and field data from a segment of a river basin containing multiple headwater sources, waste loadings, and branching streams produced good agreement between predicted and observed quality profiles.

SIMULATION OF WATER QUALITY

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INTRODUCTION

Background

The implementation of a flexible, comprehensive water plan for Texas will resolve many of the State's future water deficiency problems. Although the Texas Water Plan is based on considerable study at a reconnaissance level, it is essential that much more detailed refinement study be made prior to full plan implementation, and that this include study of water quality, particularly when evaluating various alternatives of project design, staging, and operation. The QUAL-I mathematical modeling system described in this report is designed to simulate water-quality parameters, and is one of several simulation systems being developed by the Texas Water Development Board to assist in more refined water planning and management.

Water-quality considerations are extremely broad and varied in streams, rivers, and reservoirs, and the problems associated with the effects of municipal, industrial, and agricultural wastes are of major importance. Probably the most critical aspect of waste disposal in rivers and streams is the disposition of a treated effluent from a waste process discharge. Eventually, many waste discharges enter some stream, river, or reservoir and may contribute to altering the environment. In a receiving water, it is highly desirable to make use of the waste-assimilative capacity of the water, but yet maintain water quality at a level suitable for its intended use. Thus, the capability of routing a given water-quality parameter through a stream or canal system and estimating with reasonable accuracy the waste-assimilative capacity of the system is essential to any comprehensive water resources development plan.

During the last quarter century, considerable insight has been gained into the mechanisms responsible for the transport and assimilation of various wastes discharged into streams and rivers. Particularly during the past decade, with the development of high-speed, large-memory digital computers, mathematical modeling of these transport and assimilative phenomena has been accomplished with reasonable accuracy and resolution. The importance of stream-quality models as supplemental tools in the quantitative evaluation of water-quality-oriented alternatives, such as waste treatment levels versus flow augmentation, is unquestionable.

Objective

The primary objective of this project was to develop a set of interrelated water-quality models capable of routing the following water-quality parameters through a stream subsystem:

(1) temperature,

(2) biochemical oxygen demand (BOD) and dissolved oxygen (DO), and

(3) conservative minerals.

As a necessary part of developing these models, their accuracy, dependability, and verification was to be demonstrated by application to a segment of the San Antonio River basin for which sufficient hydraulic and water quality data were available. These models, which we have designated the QUAL-I modeling system, have been completed and made operational, and their theory and description are presented in this report with a summary of verification results.[1]

Scope

Fundamental to the development of the above models were several basic requirements. These requirements were external to the actual mechanics utilized in the models, and were necessary to describe the overall structure of the models and to assure their practicality and usefulness. These requirements were as follows:

[1] A "User's Manual" that documents the computer programs and describes application of the models has also been prepared and is available for reference in the offices of the Texas Water Development Board.

1. Each of the models would be compatible with the UNIVAC 1108, CDC 6400, and RCA Sprectra 70/45 computer systems.

2. The programs would be adequately documented and flow charted.

3. Data input and output would be "user oriented".

4. Each model would be "problem oriented", physically realistic, and would result in a usable solution.

5. Each model would possess a "stand alone" capability.

6. Each model would have an "integrated system" capability so that the results of any one model could be used as input to another model, provided such feed back was required.

7. The overall program structure of the models would be "modular" in concept.

In addition to these basic model requirements, there also were several specific characteristics that the proposed models were to possess. These were as follows:

1. They would be capable of handling converging streams.

2. They would be discretized into channel elements of suitable length and variable cross-section (transverse to the channel axis).

3. They would be capable of receiving variable energy and waste inputs at selected points along the channel axis.

4. They would provide for temporal and spatial descriptions of temperature, BOD and DO, and conservative mineral variations throughout the stream or transfer system.

5. For each model, the discretized elements would be considered well mixed vertically.

6. The models would be capable of determining flow augmentation requirements under various alternative waste treatment levels (waste loadings).

7. The models would use generally accepted methods and concepts to simulate the waste-assimilative capacity of a stream or canal and to route a given waste discharge through a stream or canal system.

Organization

Development of the stream quality routing models described herein was accomplished during the period September 1969-September 1970 by W. A. White, R. J. Brandes, and Dr. F. D. Masch working in close collaboration with the Texas Water Development Board. W. A. White served as Project Director.

Collaboration with the Texas Water Development Board was facilitated by Seth Burnitt, Planning Specialist, Dr. L. F. Tischler, Systems Engineer, J. C. Moseley, Systems Engineer, and A. O. Weiss, Director of the Board's Systems Engineering Division.

Acknowledgements

MODEL DEVELOPMENT

General Considerations

The primary objective of the model development program—to provide capability for routing temperature, BOD and DO, and conservative minerals through a stream or canal system—would best be served by structuring separate models for each quality parameter and then coupling these models into an "integrated system" simulation package. Separately, one model would be capable of representing the thermal behavior of a turbulent, fully-mixed stream. A second model would describe the waste-assimilation characteristics in the stream, and a third would provide for the routing of conservative minerals such as sulfate and chloride. Linked together, the models could provide the capability for simulation of the behavior of a given quality parameter within a branching stream or canal system as well as the capability for simulating the interrelationships between the various quality parameters.

Stream Model

The degree of resolution required to determine the response of a stream or canal system to any water-quality management concept is a very difficult problem. A stream is a conglomerate of complex biological, chemical, physical, and hydraulic factors. To determine the combined effect of these various factors, mathematical models capable of representing some of the more important interrelationships between the variables have been developed.

A mathematical model is the functional representation of the response of a system or process to a given input, and is presented in a form which lends itself to solution by any acceptable method. The mathematical statement of a process consists of an input, a transfer function, and an output or response. The output from a given system is related to the input through the transfer function.

A mathematical model of a stream or canal system consists of a series of elements, each corresponding to a discrete stream or canal segment, arranged so that the output from one element becomes the input to the next. The transfer function is determined by performing a mass balance of a given water-quality parameter over a time interval, Δt, on a stream or canal segment of cross-sectional area, A, and of lengths Δx along the x-axis.

Transport

The basic equation describing the mass transport of conservative and nonconservative constituents can be written for a stream or canal segment, assuming steady-state, nonuniform flow, as

$$A\frac{\partial C}{\partial t} = \frac{\partial \left(AD_L\frac{\partial C}{\partial x}\right)}{\partial x} - \frac{\partial \left(A\bar{u}C\right)}{\partial x} \pm A\,"S" \quad (1)$$
$$\text{(i)} \qquad\qquad \text{(ii)} \qquad\qquad \text{(iii)} \qquad \text{(iv)}$$

in which

A	=	Cross-sectional area of the stream, ft^2,
C	=	Concentration of the constituent, mg/l, or temperature, $°F$,
$\bar{u}$	=	Mean velocity of the stream, ft/sec,
D_L	=	Longitudinal dispersion coefficient, ft^2/sec,
t	=	Some point in time, sec,
x	=	Some point along the longitudinal axis of the stream (x-axis), ft, and
"S"	=	Sources or sinks of a nonconservative constituent, mg/l, or temperature, $°F$.

In the equation above, term (i) represents the temporal change in concentration, term (ii) represents the transport due to longitudinal dispersion, term (iii) represents the transport due to longitudinal advection, and term (iv) represents the sources or sinks if the constituent is nonconservative.

Equation 1 can be written for a control volume or element, V_i, in the stream system as shown in Figure 1 for steady-state, nonuniform hydraulics as

$$\frac{\partial C_i}{\partial t} = \frac{\left(AD_L\frac{\partial C}{\partial x}\right)_{i+\frac{1}{2}} - \left(AD_L\frac{\partial C}{\partial x}\right)_{i-\frac{1}{2}}}{V_i} + \tag{2}$$

$$\frac{Q_{i-\frac{1}{2}}C_{i-1} - Q_{i+\frac{1}{2}}C_i \pm Q_{x_i}C_{x_i}}{V_i} + "S_i"$$

where

V_i	=	$\bar{A}_i\Delta x$ = volume of control element, ft^3,
$\bar{A}_i$	=	$\frac{1}{2}(A_{i-\frac{1}{2}} + A_{i+\frac{1}{2}})$ = mean cross-sectional area of the control volume, ft^2,
$\left(AD_L\frac{\partial C}{\partial x}\right)_{i-\frac{1}{2}}$	=	Total longitudinal dispersion of the constituent, ft^3/sec-mg/l or temperature, ft^3/sec-$°F$, on the inflow side of the control volume,
$\left(AD_L\frac{\partial C}{\partial x}\right)_{i+\frac{1}{2}}$	=	Total longitudinal dispersion of the constituent, ft^3/sec-mg/l or temperature, ft^3/sec-$°F$, on the outflow side of the control volume,

Control Volume $\overline{V}_i$

$i-1$
X

i
X

$i+1$
X

E

P

Qx_i

$Q_{i-1/2}$

ΔX

$Q_{i+1/2}$

HYDROLOGIC BALANCE

$"S"_i$

$Q_{i-1/2} C_{i-1}$

$\left(A \dfrac{D_L}{\Delta X} \dfrac{\partial C}{\partial X} \right)_{i-1/2}$

$(QxCx)_i$

$Q_{i+1/2} C_i$

$\left(A \dfrac{D_L}{\Delta X} \dfrac{\partial C}{\partial X} \right)_{i+1/2}$

MATERIAL BALANCE
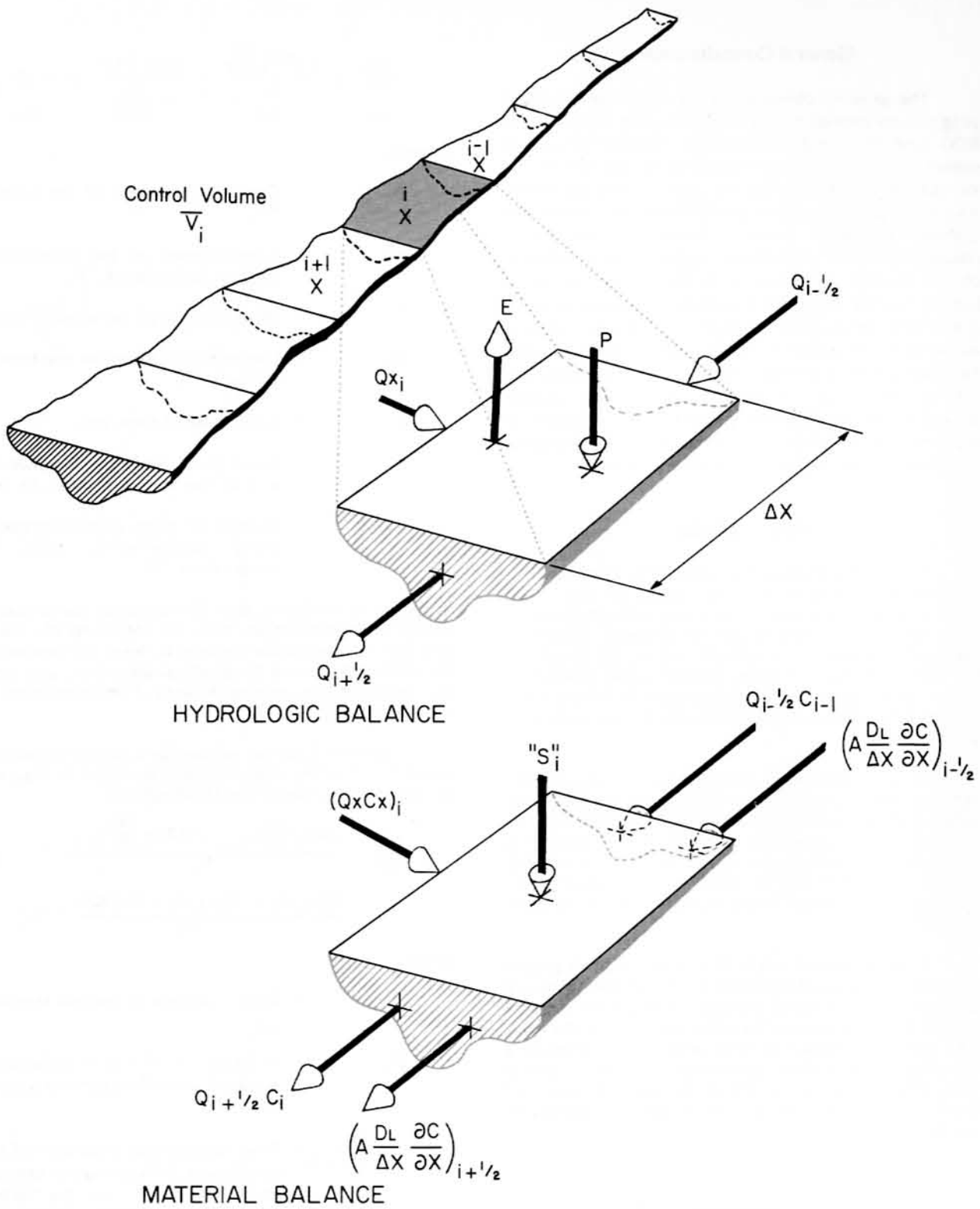
Figure 1
Discretized Stream System
After Water Resources Engineers, Inc. (1967)

$Q_{i-\frac{1}{2}}$ = Rate of flow into the control volume, ft^3/sec,

C_{i-1} = Concentration of the constituent, mg/l, or temperature, $^\circ F$, in the inflowing water,

$Q_{i+\frac{1}{2}}$ = Rate of flow out of the control volume, ft^3/sec,

C_i = Concentration of the constituent, mg/l, or temperature, $^\circ F$, in the control volume,

Qx_i = Local inflows or withdrawals, ft^3/sec,

Cx_i = Concentration of the constituent, mg/l, or temperature, $^\circ F$, in Qx, and

"S_i" = Sources or sinks of a nonconservative constituent, mg/l, or temperature, $^\circ F$.

Longitudinal Dispersion

Dispersion is basically a convective transport mechanism. The term "dispersion" is generally used for transport associated with spatially-averaged velocity variation, as opposed to "diffusion" which is reserved for transport that is associated primarily with time-averaged velocity fluctuations.

Taylor (1954) was able to derive a predictive equation for the longitudinal dispersion coefficient, D_L, in long straight pipes, as

$$D_L = 10\, r_o\, u^*, \; ft^2/sec, \tag{3}$$

where r_o is the pipe radius and u^* is the average shear velocity defined as

$$u^* = \sqrt{\mathcal{T}_o/\rho}, \; ft/sec, \tag{4}$$

where

$\mathcal{T}_o$ = Boundary shear stress, lb/ft^2, and

ρ = Mass fluid density, $lb\text{-}sec^2/ft^4$.

Some investigators have attempted to apply Taylor's expression to streamflow. However, such applications can be highly approximate, because of the difference between the geometry or velocity distributions in streamflow and those in a pipe.

Elder (1959) assumed that only the vertical velocity gradient was important in streamflow and developed an expression analogous to Taylor's expression but with a coefficient equal to 5.93:

$$D_L = 5.93\, D u^* \tag{5}$$

where D is the mean depth in ft of the stream.

Other investigators have derived similar expressions for D_L and found it to be extremely sensitive to lateral velocity profiles. Elder's expression, however, seems adequate in one-dimensional situations where the channel is not too wide. For very wide channels, Fisher (1964) has shown that half-width rather than depth is the dominant scale and therefore is important to the definition of the longitudinal dispersion coefficient. Equation 3 and 5 can be written in terms of the Manning Equation and other variables characteristic of stream channels.

As an example, for steady-state open-channel flow

$$u^* = C\sqrt{RS_e} \tag{6}$$

where C is Chezy's coefficient, R is the hydraulic radius, and S_e is the slope of the energy grade line.

Chezy's coefficient is given by

$$C = \frac{R^{1/6}}{n} \tag{7}$$

where n is the Manning roughness coefficient tabulated for different types of channels in Table 1.

Table 1.—Values of Manning's "n" Roughness Coefficient

After Henderson (1966)

Artificial Channels	n
Glass, plastic, machined metal	0.010
Dressed timber, joints flush	0.011
Sawn timber, joints uneven	0.014
Cement plaster	0.011
Concrete, steel troweled	0.012
Concrete, timber forms, unfinished	0.014
Untreated gunite	0.015-0.017
Brickwork or dressed masonry	0.014
Rubble set in cement	0.017
Earth, smooth, no weeds	0.020
Earth, some stones, and weeds	0.025

Table 1.—Values of Manning's "n" Roughness Coefficient—Continued

Natural River Channels	n
Clean and straight	0.025-0.030
Winding with pools and shoals	0.033-0.040
Very weedy, winding and overgrown	0.075-0.150
Clean straight alluvial channels	$0.031\ d^{1/6}$

$(d = D\text{-}75 \text{ size in ft})$

S_e, the slope of the energy gradient, is given by

$$S_e = \left(\frac{\bar{u}n}{1.486\ R^{2/3}}\right)^2 \tag{8}$$

where $\bar{u}$ is the mean velocity. Substituting Equations 6, 7, and 8 into Equation 5 and letting R=D for a wide channel yields the expression

$$D_L = 22.6\ n\ \bar{u}\ D^{0.833} \tag{9}$$

where

D_L = Longitudinal dispersion coefficient, ft^2/sec,

n = Manning's roughness coefficient,

$\bar{u}$ = Mean velocity, ft/sec, and

D = Mean depth, ft.

Typical values for dispersion coefficients are given in Table 2.

Table 2.—Typical Values of Dispersion Coefficients

After Gloyna (1967)

System Classification	D_L, ft^2/sec
	3×10^{-2}
Flumes and small streams	
	3
Large rivers	
	3×10^2
Estuaries	
	6×10^3

Hydrologic Balance

Assuming steady-state conditions, a hydrologic balance for the control volume can be represented by the continuity equation

$$Q_{i-\frac{1}{2}} - Q_{i+\frac{1}{2}} \pm Qx_i + P - E = 0 \tag{10}$$

where P is the precipitation rate, E is the evaporation rate, and the other terms are as previously described. For the stream model, in most instances P and E can be neglected.

The other hydraulic characteristics of a stream channel or canal can be determined from the discharge, Q, by equations of the form

$$\bar{u} = aQ^b \tag{11}$$

and

$$D = cQ^d \tag{12}$$

where a, b, c, and d, are constants. These constants usually can be determined from stage-discharge rating curves.

HEAT EXCHANGE BETWEEN A WATER BODY

AND THE ATMOSPHERE

General Considerations

In the study of the thermal behavior of a water body, it is essential to have a quantitative representation of the heat fluxes between the water surface and the atmosphere. A body of water cools by losing heat to the atmosphere; conversely, it warms by gaining heat from the atmosphere. All bodies of water cool or discharge heat to the atmosphere by back radiation, evaporation, and conduction, at the same time warming or receiving heat through short-wave solar radiation and long-wave atmospheric radiation. This chapter discusses these mechanisms and the methods with which they can be evaluated.

General Heat Budget Equation

The various mechanisms by which heat is exchanged between the water surface and the atmosphere are fairly well understood and are adequately documented in the literature by Edinger and Geyer (1965).

It is illustrative to represent the net heat flux at the water surface as shown in Figure 2. The range in magnitude of monthly average values of heat transfer included in Figure 2 are representative of northern latitudes. The expression that results from the summation of these various energy fluxes is

$$H_N = H_{sn} + H_{an} - (H_b \pm H_c + H_e) \qquad (13)$$

where

H_N = Net energy flux passing the air-water interface, Btu/ft^2-day,

H_{sn} = Net short-wave solar radiation flux passing through the interface after losses due to absorption and scattering in the atmosphere and by reflection at the interface, Btu/ft^2-day,

H_{an} = Net long-wave atmospheric radiation flux passing through the interface after reflection, Btu/ft^2-day,

H_b = Outgoing long-wave back radiation flux, Btu/ft^2-day,

H_c = Convective energy flux passing back and forth between the interface and the atmosphere, Btu/ft^2-day, and

H_e = Energy loss by evaporation Btu/ft^2-day.

Net Short-Wave Solar Radiation

The net incoming solar radiation is short-wave radiation which passes directly from the sun to the earth's surface. Its magnitude depends on: the altitude of the sun, which varies daily as well as seasonally for a fixed location on the earth; the dampening effect of scattering and absorption in the atmosphere due to cloud cover; and the reflection from the water surface.

The net amount of solar radiation which reaches the surface of the earth may be represented functionally on an hourly basis as

$$H_{sn} = H_o a_t (1 - R_s) (1 - 0.65C_L^2) \qquad (14)$$

$$\text{(i)} \quad \text{(ii)} \quad \text{(iii)} \quad \text{(iv)}$$

where

H_{sn} = Net short-wave solar radiation flux, Btu/ft^2-hour,

H_o = Amount of radiation flux reaching the earth's atmosphere, Btu/ft^2-hour,

a_t = Amospheric transmission term,

R_s = Albedo or reflection coefficient, and

C_L = Cloudiness as a fraction of sky covered.

It is appropriate for purposes of the discussion here to identify and treat separately the four components in Equation 14 as (i) extraterrestrial solar radiation, (ii) radiation scattering and absorption, (iii) reflectivity, and (iv) cloudiness.

Extraterrestrial Radiation

The short-wave solar radiation flux that strikes the earth's outer atmosphere over a given period of time is given by Water Resources Engineers, Inc. (1967) as

H_S = Short-wave solar radiation (400 to 2800 Btu/ft^2- day)

H_a = Long-wave atmospheric radiation (2400 to 3200 Btu/ft^2- day)

H_b = Long-wave back radiation (2400 to 3600 Btu/ft^2- day)

H_e = Heat loss due to evaporation (2000 to 8000 Btu/ft^2- day)

H_c = Conduction heat losses or gains (-320 to +400 Btu/ft^2-day)

H_{sr} = Reflected solar radiation (40 to 200 Btu/ft^2- day)

H_{ar} = Atmospheric reflection (70 to 120 Btu/ft^2- day)
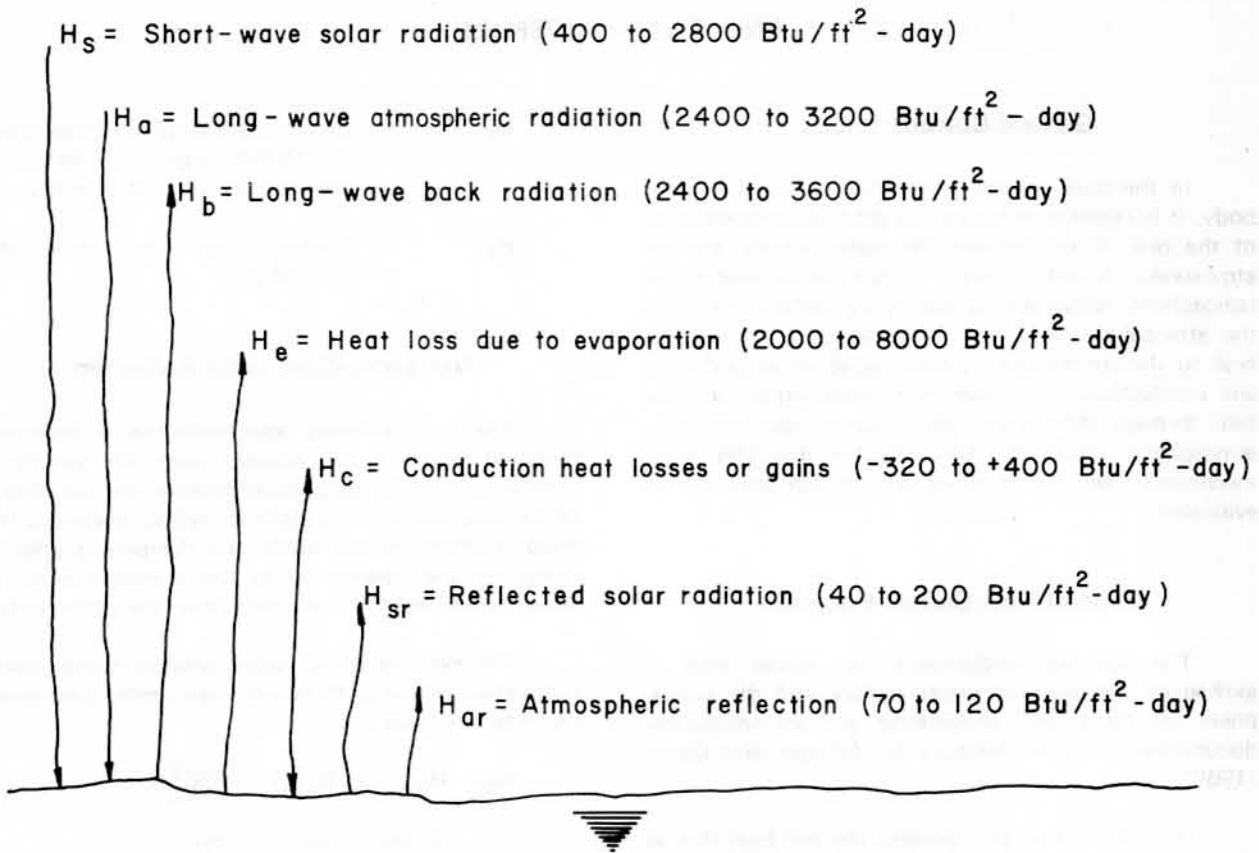
NET RATE AT WHICH HEAT CROSSES THE AIR-WATER INTERFACE:

$$H_N = \underbrace{(H_{sn} + H_{an})} - \underbrace{(H_b \pm H_c + H_e)}, \; Btu/ft^2 - day$$

Absorbed	Temperature
Radiation	Dependent
Independent	
of Temperature	

$$H_{sn} = H_s - H_{sr}$$
$$H_{an} = H_a - H_{ar}$$

Figure 2

Mechanisms of Heat Transfer Across the Air-Water Interface
After Edinger and Geyer (1965)

$$H_o = \frac{H_{sc}}{r^2} \left\{ \sin\phi \sin\delta \, (t_e - t_b) + \frac{12}{\pi} \cos\phi \cos\delta \, [\sin(\frac{\pi t_e}{12}) - \sin(\frac{\pi t_b}{12})] \right\} \Gamma \quad (15)$$

where

H_{sc} = Solar constant = 438.0 Btu/ft²-hour,

r = Normalized radius of the earth's orbit,

ϕ = Latitude of the site, degrees,

δ = Declination of the sun, degrees,

t_b, t_e = Hour angles corresponding to the beginning and end, respectively, of any time interval between sunrise and sunset, and

Γ = A correction factor for diurnal exposure to radiation flux.

Several parameters in Equation 15 requiring further definition are described by Water Resources Engineers, Inc. (1967):

Relative Earth-Sun Distance

$$r = 1.0 + 0.17 \cos[\frac{2\pi}{365}(186 - D_y)] \quad (16)$$

where D_y is the number of the day of the year (beginning January 1).

Declination

$$\delta = \frac{23.45}{180} \pi \cos[\frac{2\pi}{365}(172 - D_y)] \quad (17)$$

Hour Angles

$$t_b = ST_b - \Delta t_s + ET - 12 \quad (18)$$

and

$$t_e = ST_e - \Delta t_s + ET - 12 \quad (19)$$

where ST_b, ST_e are the standard times at the beginning and end of the time interval selected.

ET = An expression for time from a solar emphemeris which represents the difference in hours between "true solar time" and that computed on the basis of a year average. It is given for each day of the year, D_y, by

$$ET = 0.000121 - 0.12319 \sin[\frac{2\pi}{365} D_y - 0.0714] - 0.16549 \sin[\frac{4\pi}{365} D_y + 0.3088] \quad (20)$$

Δt_s = Difference between standard and local civil time in hours as determined from

$$\Delta t_s = \frac{\epsilon}{15}(L_{sm} - L_{lm}) \quad (21)$$

where

ϵ = -1 for west longitude

ϵ = +1 for east longitude

L_{sm} = Longitude of standard meridian, degrees, and

L_{lm} = Longitude of local meridian, degrees.

Diurnal Exposure

Γ = 1 when $ST_r \leq ST_b$ or $ST_e \leq ST_s$ (22)

Γ = 0 when $ST_s \leq ST_b$ or $ST_e \leq ST_r$ (23)

where ST_r and ST_s are the standard times of sunrise and sunset, respectively, as determined from

$$ST_r = 12 - \frac{12}{\pi} \arccos[\tan(\frac{\pi\phi}{180}) \tan\delta] + \Delta t_s \quad (24)$$

and

$$ST_s = 24 - ST_r + 2\Delta t_s \quad (25)$$

Radiation Scattering and Absorption

The atmospheric transmission term, a_t, is given by Water Resources Engineers, Inc. (1967) as

$$a_t = \frac{a'' + 0.5(1 - a' - d)}{1 - 0.5 R_s(1 - a' + d)} \quad (26)$$

in which a'' is the mean atmospheric transmission coefficient after scattering and absorption given by

$$a'' = \exp\left\{ -[0.465 + 0.0408 P_{wc}] [0.179 + 0.421 \exp(-0.721\theta_{am})]\theta_{am} \right\} \quad (27)$$

where θ_{am} is the optical air mass given by the expression

$$\theta_{am} = \frac{\exp(-Z/2532)}{\sin\alpha + 0.15(\frac{180\alpha}{\pi} + 3.885)^{-1.253}} \quad (28)$$

in which

Z = elevation of the site in ft, and

α = sun's altitude in radians given by

$$\alpha = \arcsin \left[\sin \frac{\pi\phi}{180} \sin \delta + \cos \frac{\pi\phi}{180} \cos \delta \cos \frac{\pi t}{12}\right] \quad (29)$$

in which t is the hour angle, described by an equation similar to Equations 18 and 19.

P_{wc} in Equation 27 is the mean daily precipitable water content in the atmosphere, given by the expression

$$P_{wc} = 0.00614 \exp (0.0489 T_d) \quad (30)$$

where T_d is the dewpoint in °F, which can be obtained from the expression

$$T_d = \ln \left[(e_a + 0.0837)/0.1001\right] /0.03 \quad (31)$$

where e_a is the water vapor pressure of the air.

The mean atmospheric coefficient, a', can also be represented by an equation of the form of Equation 27 as

$$a' = \exp \left\{-[0.465 + 0.0408 P_{wc}] \quad (32) \right.$$
$$\left. [0.129 + 0.171 \exp (-0.880 \theta am)] \theta_{am}\right\}$$

Dust attenuation of the solar radiation flux, which is represented in Equation 26 by the quantity d, varies with optical air mass, season of the year, and geographic location. Water Resources Engineers, Inc. (1967) gives a range of 0-0.13 for several locations.

Cloudiness

The dampening effect on the solar radiation flux is given by Water Resources Engineers, Inc. (1967) as

$$C_s = 1.0 - 0.65 C_L^2 \quad (33)$$

where C_L is the decimal fraction of the sky covered. Water Resources Engineers, Inc. (1967) reports that Equation 33 gives satisfactory results except for heavy overcast conditions, i.e., when C_L approaches 1.0.

Reflectivity

The reflection coefficient, R_s can be approximately computed as a function of the solar altitude, α, by Anderson's (1954) empirical formula

$$R_s = A\alpha^B \quad (34)$$

where α is in degrees, and A and B are functions of cloudiness, C_L. Values for A and B given by Anderson (1954) are shown in Table 3.

Table 3.—Empirical Coefficients for Determining R_s

After Anderson (1954)

CLOUDI-NESS C_L	0 CLEAR		0.1 - 0.5 SCAT-TERED		0.6 - 0.9 BROKEN		1.0 OVER-CAST	
	A	B	A	B	A	B	A	B
Coefficients	1.18	-0.77	2.20	-0.97	0.95	-0.75	0.35	-1.45

Long-Wave Atmospheric Radiation

The long-wave radiation emitted by the atmosphere varies directly with the moisture content of the atmosphere. Although it is primarily dependent on air temperature and humidity, it can also be affected by ozone, carbon dioxide, and possibly other materials in the atmosphere. Anderson (1954) indicated that the amount of atmospheric radiation is also significantly affected by cloud height. The amount of long-wave atmospheric radiation that is reflected is approximately a constant fraction of the incoming radiation. Anderson (1954) found this fraction to be approximately 0.03.

The net atmospheric radiation flux can be expressed as

$$H_{an} = [2.89 \times 10^{-6}] \sigma (T_a + 460)^6 (1.0 + 0.17 C_L^2)(1 - R_L) \quad (35)$$

where

H_{an} = Net long-wave atmospheric radiation flux, Btu/ft^2-hour,

σ = Stefan-Boltzman constant, 1.73×10^{-9} Btu/ft^2-hour, -°Rankine^4,

T_a = Air temperature at a level 6 feet above the water surface, °F, and

R_L = Reflectivity of the water surface for atmospheric radiation, = 0.03.

Water Surface Back Radiation

The third source of radiation transfer through the air-water interface is long-wave back radiation from the water surface, H_b, which represents a loss of heat from the water. It can be seen from Figure 2 that back radiation accounts for a substantial portion of the heat

loss from a body of water. This loss is expressed by the Stephan-Boltzman Fourth Power Radiation Law for a blackbody as

$$H_b = \sigma (T_s + 460)^4 \qquad (36)$$

where

H_b = Water surface back radiation flux, Btu/ft^2-hour, and

T_s = Water surface temperature, °F.

Evaporation

A water body also loses heat to the atmosphere by evaporation. Each pound of water that leaves as water vapor carries its latent heat of evaporation of 970 Btu. Therefore, evaporation also represents a significant loss of heat.

This heat can be expressed simply as

$$H_e = \gamma H_L E \qquad (37)$$

where

γ = Weight of the water being evaporated, lb/ft^3,

H_L = Latent heat of vaporization, Btu/lb, given by

H_L = $1084 - 0.5 T_s$, and

E = Evaporation rate, ft/hour.

The evaporation rate, E, is most often expressed as

$$E = (a + bW) (e_s - e_a) \qquad (38)$$

where

a,b, = Constants

W = Wind speed, in mph, measured 6 feet above the water surface.

e_s = Saturation vapor pressure of the air, in. of Hg, at the temperature of the water surface, as given by

e_s = $0.1001 \exp (0.03 T_s) - 0.087$ (39)

and

e_a = Water vapor pressure, in. of Hg, at a height of 6 feet above the water surface, given as

$$e_a = e_{wb} - 0.000367 P_a (T_a - T_s) (1.0 + \frac{T_{wb} - 32}{1571}) \qquad (40)$$

where

e_{wb} = Saturation vapor pressure, in. of Hg, at the wet bulb temperature from the expression

e_{wb} = $0.1001 \exp (0.03 T_{wb}) - 0.0837$ (41)

P_a = Local barometric pressure, in. of Hg, and

T_{wb} = Wet bulb temperature, °F.

The literature contains a wide range of values for the evaporation constants a and b. Roesner (1969) reports that a good average value of a would be 6.8×10^{-4} ft/hour-in. of Hg, while b would best be represented by 2.7×10^{-4} ft/hour-in. of Hg-mph.

Conduction

Heat that is transferred between the water and the atmosphere due to a temperature difference between the two phases is normally called conduction. Using the fact that transfer by conduction is a function of the same variables as evaporation, it is possible to arrive at a proportionality between heat conduction and heat loss by evaporation. This proportionality, known as Bowen's ratio, is expressed as

$$B = \frac{H_c}{H_e} = C_B \left[\frac{T_s - T_a}{e_s - e_a} \right] \frac{P_a}{29.92} \qquad (42)$$

where C_B is a coefficient $\cong 0.01$.

By using Bowen's ratio, the rate of heat loss to the atmosphere by heat conduction, H_c, can be defined as

$$H_c = \gamma H_L (a+bW) (0.01 \frac{P_a}{29.92}) (T_s - T_a) \qquad (43)$$

Energy Budget and Internal Mixing

A stream may be considered completely mixed in its transverse section if the heat exchange with the environment affects the water body practically instantaneously over its entire depth. Wunderlich (1969) found that the internal turbulence in many streams was sufficient to assure practically instantaneous heat distribution from the surface downward. However, as velocities decrease and depths increase, this assumption becomes less valid.

- 13 -

Thus, assuming complete mixing, Equation 1 can be written with a source term for temperature as

$$A\frac{\partial C}{\partial t} = \frac{\partial(A D_L \frac{\partial C}{\partial x})}{\partial x} - \frac{\partial (A \bar{u} C)}{\partial x} + \frac{A}{\gamma c_p} \text{"}S_H\text{"} \qquad (44)$$

where

C = Temperature, $°F$,

γ = Weight of water, 62.4 lb/ft^3,

c_p = Specific heat of water, 1.0 Btu/lb-$°F$, and

"S_H"   = Heat source term, Btu/ft^3-hour.

The parameters γ and c_p can be considered constant for practical applications.

The source term "S_H", which has units of Btu/ft^3-hour, accounts for internal heat generation and all heat transferred across the system boundaries, i.e., heat transferred across the air-water interface and heat conducted across the mud-water interface. In the absence of ground-water flow, heat is transported across the mud-water interface only by molecular conduction which is relatively insignificant in comparison to surface heat exchange.

From Equation 13 the net radiation flux at the water surface is H_N having units Btu/ft^2-hour. For a stream of length Δx and mean surface width W, the total rate of heat transfer across the air-water interface is $H_N \Delta x$ W. This heat is distributed uniformly throughout the underlying volume of $\bar{A}\Delta x$, where $\bar{A}$ is the mean cross-sectional area of the reach. Thus, the net rate of heat gain or loss per unit volume of water, "S_H", is computed as

$$\text{"}S_H\text{"} = \frac{H_N \Delta x \, W}{\bar{A}\Delta x} = \frac{H_N}{D} \qquad (45)$$

where $D = \bar{A}/W$ and is the hydraulic depth of the stream. Substituting Equation 45 back into Equation 44 yields the basic one-dimensional heat transport equation

$$A\frac{\partial C}{\partial t} = \frac{\partial(A D_L \frac{\partial C}{\partial x})}{\partial x} - \frac{\partial (A u C)}{\partial x} \pm \frac{A H_N}{\gamma c_p D} \qquad (46)$$

BIOCHEMICAL OXYGEN DEMAND AND
DISSOLVED OXYGEN BALANCE

General Considerations

The most important consideration in determining the waste-assimilative capacity of a stream is its ability to maintain an adequate dissolved-oxygen concentration. Dissolved-oxygen concentrations in streams are controlled by atmospheric reaeration, photosynthesis, plant and animal respiration, benthal demand, biochemical oxygen demand, nitrification, salinity, and temperature, among other factors.

The most accurate oxygen balance would consider all significant factors. However, many of the factors are very difficult, if not impossible, to define accurately; and unless unusual conditions are present, fairly reliable predictions of the "self-purification process" of a water body can be obtained through simulation of the simultaneous processes of reaeration (natural or artificial) and deoxygenation as measured by the biochemical oxygen demand.

An extremely significant factor in a stream's oxygen resources is the seasonal temperature variation. During the winter, when the water temperature is lowest, the solubility of oxygen, and thus its availability for biological respiration, is greatest. Unfortunately, the cooler temperatures retard the rate of biological activity so that respiration is at a minimum. Conversely, at high summer water temperatures oxygen solubility and availability are lowest and biological respiration proceeds at its maximum rate, thus severely depleting the oxygen resources of the stream if significant amounts of biodegradable material are present. Generally speaking, the latter condition is the critical one in terms of the aquatic environment and the amount of biologically degradable material which the stream can assimilate.

Reaeration

One of the major phenomena contributing to the biochemical oxidation in waters containing degradable materials is atmospheric reaeration. Many theories have been proposed and a large number of techniques and equations have been used to estimate the reaeration coefficient, K_2; however, there is no universally accepted method for doing so.

It is generally accepted that the reaeration process can be expressed as

$$\frac{\partial D}{\partial t} = -K_2 (C_s - C_t) \qquad (47)$$

where

D $= (C_s - C_t) =$ oxygen deficit, mg/l,

K_2 $=$ Reaeration coefficient, 1/days,

C_s $=$ Solubility of oxygen in water, mg/l, and

C_t $=$ Existing concentration of oxygen at time t, mg/l.

Solubility of Oxygen in Water

The solubility of oxygen in water is primarily dependent upon temperature, pressure, and the concentration of dissolved salts. At standard pressure (29.92 in. of Hg) the solubility of oxygen in water can be given as

$$C_s = 24.89 - 0.426T + 0.00373T^2 - 0.0000133T^3 \qquad (48)$$

where T = temperature of water, in °F. According to the American Public Health Association, Inc. (1965), C_s can be corrected for a given barometric pressure other than standard pressure by the equation

$$C_s = C_s \frac{P_a - e_s}{29.92 - e_s} \qquad (49)$$

where

P_a $=$ Barometric pressure, in. of Hg, and

e_s $=$ Saturated water vapor pressure at the temperature of the water surface, in. of Hg;

and for elevation less than 3,000 feet by

$$C_s' = C_s \frac{P_a}{29.92} \qquad (50)$$

For water temperatures above 60°F, the American Public Health Association, Inc. (1965) indicates that the solubility of oxygen in water decreases by approximately 0.008 mg/l per 100 mg/l of chloride present.

Reaeration Coefficients

The reaeration process is significantly influenced by temperature, stream geometry, and stream hydraulics. Based on extensive experimental investigations, the value of the reaeration coefficient has been found by Eckenfelder and O'Connor (1961) to vary with temperature as

$$K_2^T = K_2^{20} (1.047)^{20-T} \qquad (51)$$

where T is the temperature of the water in °C.

Numerous equations have been developed to compute reaeration coefficients based on stream geometry and stream characteristics. Some of these are discussed below and are in terms of the Naperian base (e).

Churchill, Elmore, and Buckingham (1962)

This investigation was based on probably the most extensive and accurate measurements of stream reaeration available and produced the following expression for K_2 at 20°C (68°F):

$$K_2^{20} = 5.026 \, \bar{u}^{0.969} \, D^{-1.673} \times 2.31 \qquad (52)$$

where

$\bar{u}$	=	Average velocity in the stream, ft/sec,
D	=	Average depth of the stream, ft, and
K_2	=	Reaeration coefficient, 1/days.

O'Conner and Dobbins (1958)

These investigators proposed equations based on the turbulent characteristics of a stream as follows:

For streams displaying low velocities and isotropic conditions

$$K_2^{20} = \frac{(D_m \, \bar{u})^{0.5}}{D^{1.5}} \qquad (53)$$

For streams displaying high velocities and non-isotropic conditions

$$K_2^{20} = \frac{480 \, D_m^{0.5} \, S_o^{0.25}}{D^{1.25}} \times 2.31 \qquad (54)$$

where

S_o	=	Slope of the streambed,
D	=	Mean stream depth, ft,
$\bar{u}$	=	Mean velocity, ft/day,
K_2	=	Reaeration coefficient, 1/days,

and where D_m is the molecular diffusion coefficient (ft^2/day) which can be computed by

$$D_m = 1.91 \times 10^{-3} (1.037)^{T-20} \qquad (55)$$

Isotropic conditions are satisfied when Chezy's coefficient is greater than 17, and non-isotropic for values less than 17. O'Conner and Dobbins (1958) have shown that Equation 53 is generally applicable for most cases.

Owens, Edwards, and Gibbs (1964)

For streams with a velocity variation range from 0.1 to 5.0 ft/sec and depths from 0.4 to 11.0 ft:

$$K_2^{20} = 9.4 \, \bar{u}^{0.67} / D^{1.85} \times 2.31 \qquad (56)$$

where

$\bar{u}$	=	Mean velocity, ft/sec,
D	=	Mean depth, ft, and
K_2	=	Reaeration coefficient, 1/days.

For streams with a velocity variation range from 0.1 to 1.8 ft/sec and depths from 0.4 to 11.0 ft:

$$K_2^{20} = 10.1 \, \bar{u}^{-0.73} / D^{1.75} \times 2.31 \qquad (57)$$

Thackston and Krenkel (1966)

This investigation included several rivers in the Tennessee Valley Authority system and resulted in the following equation for K_2 at 20°C:

$$K_2^{20} = 10.8 (1 + F^{0.5}) \frac{u^*}{D} \times 2.31 \qquad (58)$$

where F is the Froude number which can be computed by

$$F = \frac{u^*}{\sqrt{gD}} \qquad (59)$$

and u^* is the shear velocity, ft/sec, which can be computed by

$$u^* = \sqrt{DS_{e}g} \qquad (60)$$

where

D	=	Mean depth, ft,
g	=	Acceleration of gravity, ft/sec², and
S_e	=	Slope of the energy gradient.

Langbien and Durum (1967)

$$K_2^{20} = 3.3\bar{u} / D^{1.33} \tag{61}$$

where

$\bar{u}$ = Mean velocity, ft/sec,

D = Mean depth, ft, and

K_2 = Reaeration coefficient, 1/days.

Numerous other equations have been proposed for computing reaeration coefficients; however, for the most part they are extensions of a basic equation given by Streeter and Phelps (1925):

$$K_2 = \frac{c\,\bar{u}^n}{D^2} \tag{62}$$

where

$\bar{u}$ = Mean stream velocity, ft/sec,

D = Mean stream depth, ft, and

c,n = Constants for a particular stream in question.

Biochemical Oxygen Demand

The rate of oxygen utilization (deoxygenation) due to biological activity is of particular importance in streams. When the concentration of dissolved oxygen in a stream is at the saturation level (this is normally the case when there is no oxygen demand exerted and no significant amount of algae present), the rate of transfer of oxygen across the air-water interface is zero. However, as oxygen demand is exerted the dissolved-oxygen concentration is reduced below the saturation level and oxygen is transferred across the air-water interface and into the stream by reaeration. The larger the oxygen deficit becomes, the greater the reaeration rate. At some point downstream from the introduction of a load of biologically degradable material, the reaeration rate balances the rate at which the oxygen is being utilized. This is the minimum point in the oxygen sag curve as illustrated in the classical situation in Figure 3. It is essential to know this minimum concentration of dissolved oxygen and where it occurs in order that effluent quality standards may be set, or to determine how much flow augmentation may be required if biodegradable material in the effluent cannot be removed to a sufficient extent to meet a target dissolved-oxygen level.

The oxygen demand in polluted waters is exerted by three classes of materials: (1) carbonaceous organic material which is usable as a carbon and/or energy source by aerobic micro-organisms; (2) oxidizable nitrogen which can be used as an energy source by specific genera of bacteria; and (3) some reduced inorganic compounds which will react with dissolved molecular oxygen. The biochemical oxygen demand test is used to estimate the biological oxygen demand of a waste or stream sample. Compounds in category (3) above are excluded from the test by normal sample preparation procedures described by the American Public Health Association, Inc. (1965).

The rate of oxygen utilization due to biochemical oxygen demand (BOD) can generally be expressed as a first-order bio-kinetic reaction:

$$\frac{\partial L}{\partial t} = -K_1 L_t \tag{63}$$

where

K_1 = BOD rate constant, *base (e)*, mg/l, and

L_t = Concentration of BOD (ultimate), mg/l, at time t.

The BOD rate constant, K_1, has traditionally been determined in the laboratory; however, it is quite obvious that the rate of BOD exertion in a laboratory bottle is not necessarily the same as in a natural stream. The value of K_1 not only depends on the nature of the waste material, but is affected by mixing, deposition, and adsorption.

Several important limitations on the use of the conventional BOD test as a measure of the total demand on the oxygen resources of a stream must be considered. First, the BOD test has no direct stoichiometric relationship to the organic content of a water sample or the biological population in the sample. Thus, BOD data frequently cannot be correlated to known concentrations of organic substances. The BOD test is usually run for 5 days, a time period with no particular physical or biological significance. It is commonly presumed that using this time period will eliminate the oxygen demand of nitrogenous compounds, but this is frequently not the case, particularly for natural stream samples.

The 5-day BOD (BOD_5) test alone gives no clue to the total organic content of a water sample or the total oxygen demand. If the BOD test is run at varied intervals (1 day, 2 days, 3 days, etc.), so that the reaction rate constant can be obtained, then the ultimate carbonaceous BOD can be determined. However, even this procedure will not give a clue to the total oxygen demand of the sample. Reduced nitrogenous compounds, both organic and inorganic, will also exert an oxygen demand as they are converted to nitrate by certain bacteria. Specialized techniques (21-day BOD tests, chemical inhibition) must be used to separate the
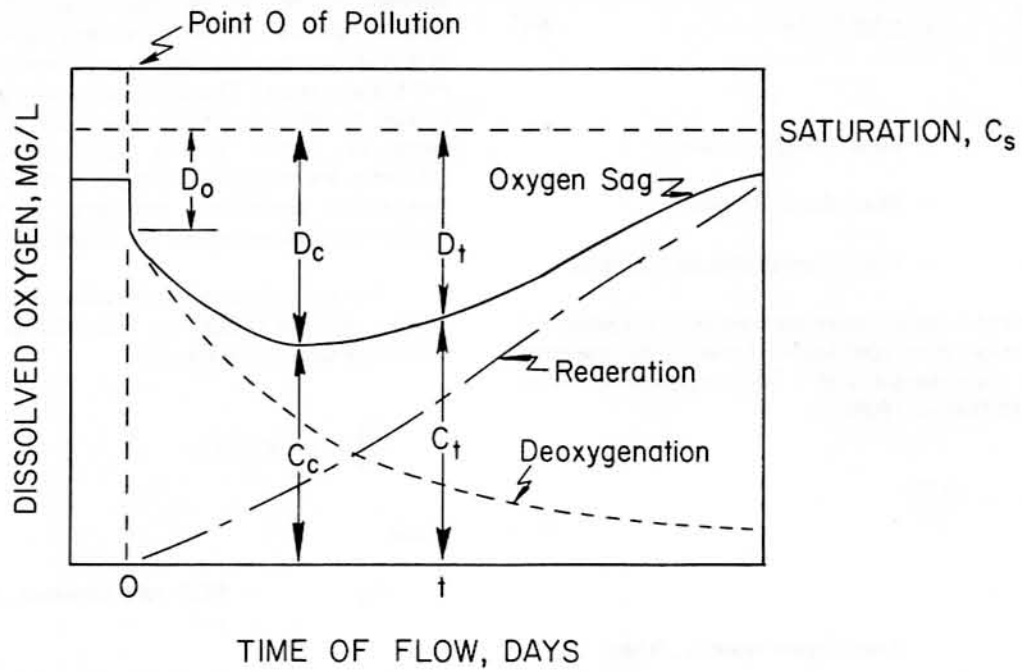
Reaeration: $\frac{\partial C}{\partial t} = K_2 (C_s - C_t)$

Deoxygenation: $\frac{\partial L}{\partial t} = -(K_1 + K_3) L$

D_c = Critical oxygen deficit

D_o = Initial oxygen deficit

D_t = Deficit at time t

C_t = Oxygen concentration at time t

C_c = Critical oxygen concentration

Figure 3
The Oxygen Sag Curve and Components
After Eckenfelder and O'Conner (1961)

carbonaceous and nitrogeneous oxygen demands so that each may be quantified. As mentioned above, it has often been assumed that the 5-day BOD test would exclude the nitrogeneous demand due to the slow growth rates of the nitrifying bacteria. This assumption may be valid for raw sewage but is probably the exception rather than the rule for natural streams.

All of the above problems complicate the use of data from BOD_5 tests in modeling. However, most historical data are of this type, and these data must be utilized until the determination of more representative water-quality parameters becomes widespread. To use the historical data from BOD_5 tests for modeling purposes, it is necessary to select a value of the reaction constant, K_1 which is felt to be best for the given condition.

Thomas (1948) gives values of

$$K_1 \quad = 0.06 \text{ to } 0.36 \text{ per day}$$

for most streams handling non-toxic wastes in warm weather. K_1 is also affected by temperature and has been found to vary as

$$K_1^T = K_1^{20} (1.075)^{(20-T)} \tag{64}$$

where T is the temperature of the water in $°C$. Thomas (1948) also proposed the concept of a K_3 value to account for the rate of BOD removal by deposition, and gives values for most American streams of

$$K_3 \quad = -0.36 \text{ to } 0.36 \text{ per day.}$$

BOD, DO Balance, and Internal Mixing

As was done with temperature, the assumption is made that the transverse section of the stream is completely mixed. The validity of this assumption is probably more questionable than the one made for temperature. It is well known that in many instances wastewaters are not thoroughly mixed with the receiving water stream for many miles downstream of the point of release.

Nevertheless, assuming complete mixing, Equation 1 can be written with a source or sink term for oxygen as

$$A\frac{\partial C}{\partial t} = \frac{\partial (ADL\frac{\partial C}{\partial x})}{\partial x} - \frac{\partial (A\bar{u}C)}{\partial x} \pm A \text{ "}S_{DO}\text{"} \tag{65}$$

where

$$\text{"}S_{DO}\text{"} = K_2(C_s - C) - (K_1 + K_3) L, \text{ mg/l-sec, and}$$

$$C \quad = \text{Concentration of dissolved oxygen, mg/l;} \tag{66}$$

and for the biochemical oxygen demand as

$$A\frac{\partial L}{\partial t} = \frac{\partial (ADL\frac{\partial L}{\partial x})}{\partial x} - \frac{\partial (A\bar{u} L)}{\partial x} \pm A \text{ "}S_L\text{"} \tag{67}$$

where

$$\text{"}S_L\text{"} = -(K_1 + K_3) L, \text{ mg/l-sec, and} \tag{68}$$

$$L \quad = \text{Concentration of BOD (ultimate), mg/l.}$$

Flow Augmentation

When environmental conditions are such that the dissolved-oxygen concentration in a stream drops below some required target level, flow augmentation may be desirable. The amount of augmentation water required to bring dissolved-oxygen concentrations up to required standards cannot be computed by an exact functional relationship; however, a good approximation can be given by

$$DO_R \quad = DO_T - C_c \tag{69}$$

and

$$Q_R \quad = Q_c \frac{DO_R}{DO_T} \tag{70}$$

where

DO_R = Dissolved-oxygen concentration required to meet target conditions, mg/l,

DO_T = Some required target level of dissolved oxygen, mg/l,

C_c = Minimum dissolved-oxygen concentration (critical level) in the oxygen sag curve, mg/l,

Q_R = Amount of flow augmentation required, cfs, and

Q_c = Flow at the critical point in the oxygen sag curve, cfs.

General Considerations

A conservative mineral is one that is assumed to have no sources or sinks other than local inflows or diversions. The mineral is not significantly affected by changes in temperature or any chemical, biological, or other process. Water-quality parameters that are considered as conservative include total dissolved solids, chloride, and sulfate.

Total dissolved solids (TDS) represents all organic and inorganic matter in water. In most cases the inorganic materials are present in quantities greatly in excess of the organic materials present. In such cases, total dissolved solids can be considered as conservative minerals.

Conservative Mineral Balance

A typical example of the distribution of a conservative mineral within a stream system is illustrated in Figure 4. It can be seen here that the concentration varies with the stream discharge, Q. Thus, routing a conservative mineral requires no more than a material balance. Therefore, Equation 1 without a source or sink term is sufficient to describe the behavior of a conservative mineral within a stream or canal system and it is given as

$$A\frac{\partial C}{\partial t} = \frac{\partial (AD_L\frac{\partial C}{\partial x})}{\partial x} - \frac{\partial (A\bar{u} C)}{\partial x} \qquad (71)$$

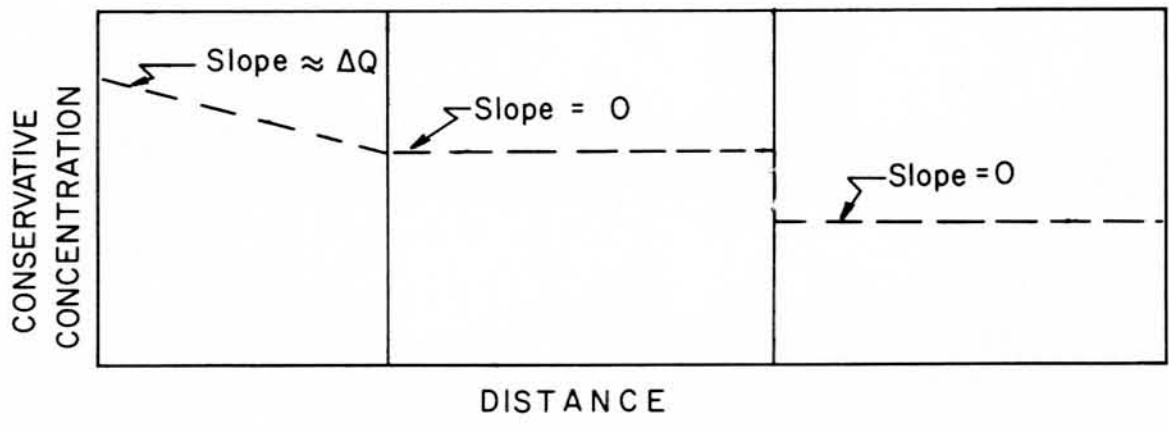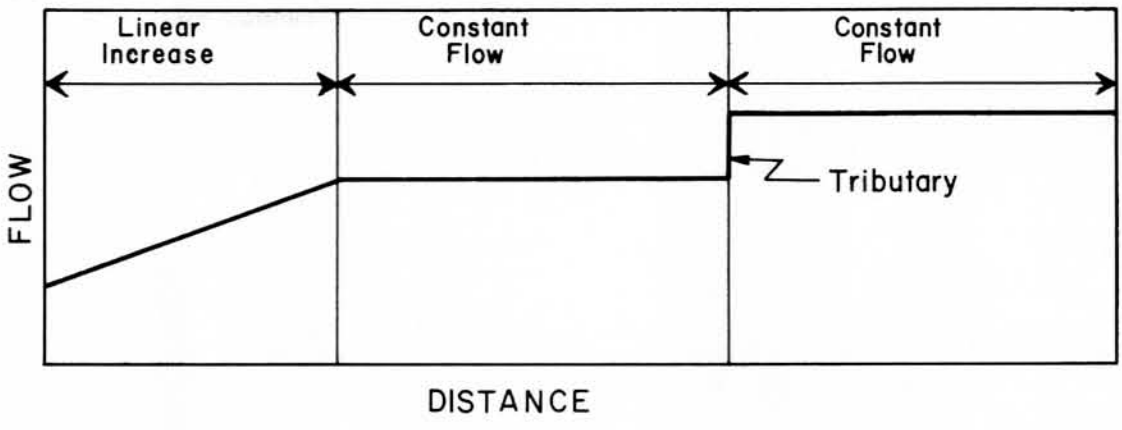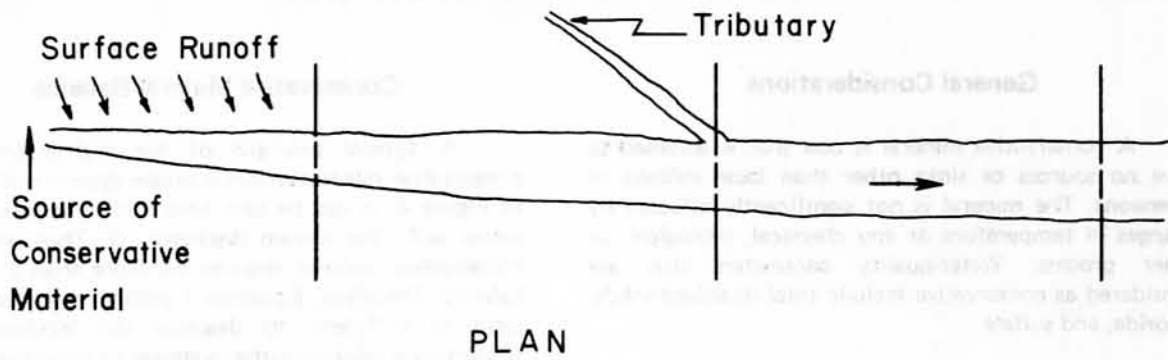where

C = Concentration of conservative mineral, mg/l.

Figure 4
Effect of Flow on Conservative Mineral

General Considerations

Since it is not possible to obtain an analytical solution to Equation 1 under most prototype situations, a finite-difference method is used for its solution. Three basic finite-difference methods are applicable to the advection-dispersion equation under consideration: (1) an explicit or forward difference scheme, (2) an implicit or backward difference scheme, or (3) a combination of the first two (which results in an implicit scheme also). An excellent discussion of the various finite-difference methods is given by Smith (1966). In general, explicit schemes have the advantage of computational simplicity but are severely restricted by instability requirements in their solution. Instability refers to the ratio of the exponentially increasing divergence between the numerical solution and the true solution of the difference equation under consideration. To avoid such instabilities, a limit must be placed on the size of the time step used in advancing the solution. Implicit methods, on the other hand, are unconditionally stable for any size time step, but require more complicated methods of solution, hence more computer time.

The general basis of a finite-difference scheme is to find the value of a variable (e.g., temperature, BOD and DO, or conservative mineral) as a function of space at a time step $n+1$ when its spatial distribution at the nth time step is known. Time step zero corresponds to the initial condition. Forward difference schemes are characterized by the fact that all spatial derivatives $(\partial/\partial x)$ are approximated in difference form at time level n only, whereas in backward difference schemes, all spatial derivatives are written in difference form at time level $n+1$ or they are averaged over both time steps. The classical implicit backward difference approximation (all spatial derivatives are written at time step $n+1$) will be used to formulate the equations in the following discussion.

Formulation

The finite difference scheme will be formulated by considering the constituent, C, at four points in the nemonic scheme as shown in Figure 5.

$$n + 1 : t = t + \Delta t$$

$$n \quad : t = t$$

Classical Implicit Nodal Scheme

Three points are required at time $n+1$ to approximate the spatial derivatives. The temporal derivative is approximated at distance step i.

The form of the differential equation used in the numerical solution of the transport equation is similar to Equation 2, except written in a slightly different form

$$\frac{\partial C_i}{\partial t} = \frac{(AD_L\frac{\partial C}{\partial x})_i - (AD_L\frac{\partial C}{\partial x})_{i-1}}{V_i} + \frac{Q_{i-1} C_{i-1} - Q_i C_i \pm Qx_i Cx_i}{V_i} \pm \text{"}S_i\text{"} \quad (72)$$

The resulting finite-difference form of Equation 72 is

$$-\left\{(AD_L)_{i-1} \frac{\Delta t}{V_i\Delta x} + Q_{i-1} \frac{\Delta t}{V_i}\right\}C_{i-1}^{n+1} + \left\{1.0 + [(AD_L)_i + (AD_L)_{i-1}] \frac{\Delta t}{V_i\Delta x} + Q_i \frac{\Delta t}{V_i}\right\}C_i^{n+1} - \left\{(AD_L) \cdot (AD_L)_i \frac{\Delta t}{V_i\Delta x}\right\} C_{i+1}^{n+1} = Z_i \quad (73)$$

where

$$Z_i = C_i^n + \Delta t \text{"}S_i\text{"} + \Delta t\, Qx_i\, Cx_i, \quad (74)$$

$$V_i = \tfrac{1}{2} (A_i + A_{i-1}) \Delta x = \text{volume of element } i, \quad (75)$$

and all other terms are as previously defined. All of the values on the right-hand side of Equation 73 are known at time step n and all of those on the left-hand side are unknowns at time step $n+1$. The coefficients on the left-hand side can be given as

$$a_i = -(AD_L)_{i-1} \frac{\Delta t}{V_i\Delta x} - Q_{i-1} \frac{\Delta t}{V_i}, \quad (76)$$

$$b_i = 1.0 + [\,(AD_L)_{i-1} + (AD_L)_i\,]\frac{\Delta t}{V_i\Delta x} + Q_i \frac{\Delta t}{V_i}, \text{ and} \quad (77)$$

$$c_i = -(AD_l)_i \frac{\Delta t}{V_i\Delta x}. \quad (78)$$

Method of Solution

Equation 73 represents a tridiagonal set of linear equations for the solution of C_i^{n+1} for all i's. They can be represented in matrix form as

$$\begin{bmatrix} b_1 & c_1 & & & & & \\ a_2 & b_2 & c_2 & & & & \\ & a_3 & b_3 & c_3 & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & a_i & b_i & c_i & \\ & & & & \cdot & \cdot & \cdot \\ & & & & & a_{l-1} & b_{l-1} & c_{l-1} \\ & & & & & & a_l & b_l \end{bmatrix} \times \begin{bmatrix} C_1^{n+1} \\ C_2^{n+1} \\ C_3^{n+1} \\ \vdots \\ C_i^{n+1} \\ \vdots \\ C_{l-1}^{n+1} \\ C_l^{n+1} \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ \vdots \\ Z_i \\ \vdots \\ Z_{l-1} \\ Z_l \end{bmatrix} \quad (79)$$

An efficient method that readily lends itself to a computer solution of the set of Equations 79 is described by Ralston and Wilf (1960). The method of solution is as follows:

(1) Divide through the first equation in 79 by b_1 to obtain

$$C_1^{n+1} + W_1 C_2^{n+1} = G_1 \qquad (80)$$

where

$$W_1 = c_1/b_1 \text{ and } G_1 = Z_1/b_1. \qquad (81)$$

(2) Combine Equation 80 and the first equation in 79 to eliminate a_2 and the result is

$$C_2^{n+1} + W_2 C_3^{n+1} = G_2 \qquad (82)$$

where

$$W_2 = \frac{c_2}{b_1 - a_2 W_1} \text{ and } G_2 = \frac{Z_2 - a_2 G_1}{b_2 - a_2 W_1}. \qquad (83)$$

(3) Combine Equation 82 and the third equation in 79 to eliminate a_3 and the result is

$$C_3^{n+1} + W_3 C_4^{n+1} = G_3 \qquad (84)$$

where

$$W_3 = \frac{c_3}{b_3 - a_3 W_2} \text{ and } G_3 = \frac{Z_3 - a_3 G_2}{b_3 - a_3 W_2}. \qquad (85)$$

(4) Proceed through the equations, eliminating a_i and storing the values of W_i and G_i given by

$$W_i = \frac{c_i}{b_i - a_i W_{i-1}}, \ i = 2, 3, \ldots, I \qquad (86)$$

and

$$G_i = \frac{Z_i - a_i G_{i-1}}{b_i - a_i W_{i-1}}, \ i = 2, 3, \ldots, I. \qquad (87)$$

(5) The last equation is solved for C_I^{n+1} by

$$C_I^{n+1} = G_I \qquad (88)$$

(6) Solve for $C_{I-1}^{n+1}, C_{I-2}^{n+1}, \ldots, C_1^{n+1}$ by back substitution:

$$C_i^{n+1} = G_i - W_i C_{i+1}^{n+1}, i = I-1, I-2, \ldots, 1. \qquad (89)$$

Boundary Conditions

Upstream

For most fresh-water streams, transport is unidirectional in nature, i.e., there is no significant transport upstream. Therefore, the concentration at some point just upstream from the upper end of the stream reach of interest can be used as the boundary condition. Hence, Z_1 in Equation 73 is taken as

$$Z_1 = C_1^n + \frac{Qx_1 Cx_1}{V_1} \pm "S_1" + a_1 C_o^{n+1} \qquad (90)$$

where C_o^{n+1} is the boundary condition.

Downstream

For the boundary condition at the downstream end of the system it is possible to assume a fictitious boundary condition at a point only slightly downstream from the lower end of the stream reach of interest. This is possible because the magnitudes of D_L and u in virtually all fresh-water streams are such that the downstream boundary condition has very little effect on the water upstream. Then, one can let

$$C_{I+1}^{n+1} = C_I^{n+1} \qquad (91)$$

where C_{I+1}^{n+1} is the concentration just downstream from the end of the system.

Initial Conditions

In most cases of practical interest, one is concerned with the quasi steady-state concentration profiles which result from continuous, but periodically varying, inputs. Theoretically, an infinite time is required for the steady-state conditions to develop from any given initial conditions. However, what is essentially the steady-state condition is reached in a time equal to the time of flow (travel time) in the stream reach. Also, the steady-state concentration profiles are essentially independent of the initial conditions. Physically, this means that by the time the front of a load being put in at x=0 reaches the downstream end of the reach, most of the original water has been flushed out of the reach and the concentration profiles are dependent only on the inputs. Therefore, initial conditions can be a matter of choice.

Selection of Time and Distance Steps

In using the implicit finite-difference scheme it is necessary to select values for Δt and Δx. Although there is no restriction upon Δt and Δx for stability of the solution of Equation 73, Stone and Brian (1963) suggest that to guarantee good accuracy

$$\bar{u}\frac{\Delta t}{\Delta x} \leq 1 \qquad (92)$$

However, in the interest of economical solutions, larger time steps can be used if no noticeable deviations in the results are detected. In general, a value of Δx will be chosen, then the value of Δt determined from Equation 92. The value of $\bar{u}$ should be the maximum to be expected in the equations.

General Considerations

The measure of a model's usefulness is its ability to simulate the behavior of a prototype system within reasonable limits of accuracy with a minimal amount of time and effort on the user's part. The ability of the models, described herein, to simulate a prototype system will depend upon a number of factors inherent to the modeling process:

(1)   the degree to which spatial detail is provided in the model for describing the geometry of the prototype system,

(2)   the degree to which temporal detail is provided in the model for describing time-varying inputs and responses,

(3)   the ability of the model to functionally represent the physical, biological, and chemical processes which are characteristic of the prototype system,

(4)   the accuracy and reliability of the computational techniques which transform given inputs through a transfer function to obtain a response in the system, and

(5)   the amount and reliability of the basic data that are required to describe the characteristics of the prototype system and all inputs that are essential.

Temporal and spatial detail in the models developed in this study are, by their very nature, extremely flexible. A stream system can be segmented along its longitudinal axis to provide any degree of resolution that is desired, and time intervals can be selected so that they are compatible with time-variant inputs and responses. With an increase in temporal and spatial detail, however, comes the additional expense of increased computer time.

Another limitation of the models is the one-dimensionality in a mathematical sense of their various transfer mechanisms. This means that the user is constrained to fully-mixed (both vertically and laterally) streams. However, for most cases of interest this limitation can be tolerated.

The accuracy and reliability of the basic data have always been a problem in model verification and application and probably always will be. In preparing data for use as input to a model, it is synthesized, statisticized, averaged, extrapolated, and otherwise operated on to render it into some usable form desired by the user of the model. Any one or all of these data

manipulations can introduce uncertainties which cannot always be anticipated. Thus, the user must give this consideration when interpreting the results obtained from the model application.

Selection of a Test Case

Part of the San Antonio River basin was selected for testing the QUAL-I modeling system. This basin comprises an area of more than 4,100 square miles in south-central Texas and includes parts of two physiographic sections as shown in Figure 6. These are the Edwards Plateau section of the Great Plains province and the West Gulf Coastal Plain section of the Coastal Plain province. In the upper part of the basin the principal stream is the Medina River, which rises in northwestern Bandera County, flows southeastward across the Edwards Plateau, and joins the San Antonio River about 15 miles south of the city of San Antonio as shown in Figure 7.

The mainstem of the San Antonio River rises in the city of San Antonio near the center of Bexar County, flows southeastward across the West Gulf Coastal Plain, and joins the Guadalupe River about 11 miles upstream from San Antonio Bay, an estuary of the Gulf of Mexico. Of the several tributaries, the principle one is Cibolo Creek, which rises in Kendall County in the Edwards Plateau section, flows southeastward across the Balcones Escarpment and West Gulf Coastal Plain section, and joins the San Antonio River in Karnes County.

During a summer low-flow period, June 16-19, 1969, a water-quality study in the San Antonio River basin was carried out by the U.S. Geological Survey in cooperation with the Texas Water Development Board, and the results have been described by Rawson (1970). Quality and streamflow data were collected in that study at six sites on the San Antonio River and at one site on Cibolo Creek, as shown in Figure 7. The quality data consisted of temperature, dissolved oxygen, biochemical oxygen demand (5-day), the "nitrogen series", and selected conservative minerals and dissolved salts. The information given in Rawson (1970) has been supplemented with data on waste discharges and stream withdrawals supplied by the San Antonio River Authority and the Alamo Area Council of Governments, and local climatological data supplied by the U.S. Weather Bureau.

The part of the San Antonio River basin actually used in model application and verification is shown schematically in Figure 8. This segment consists of the
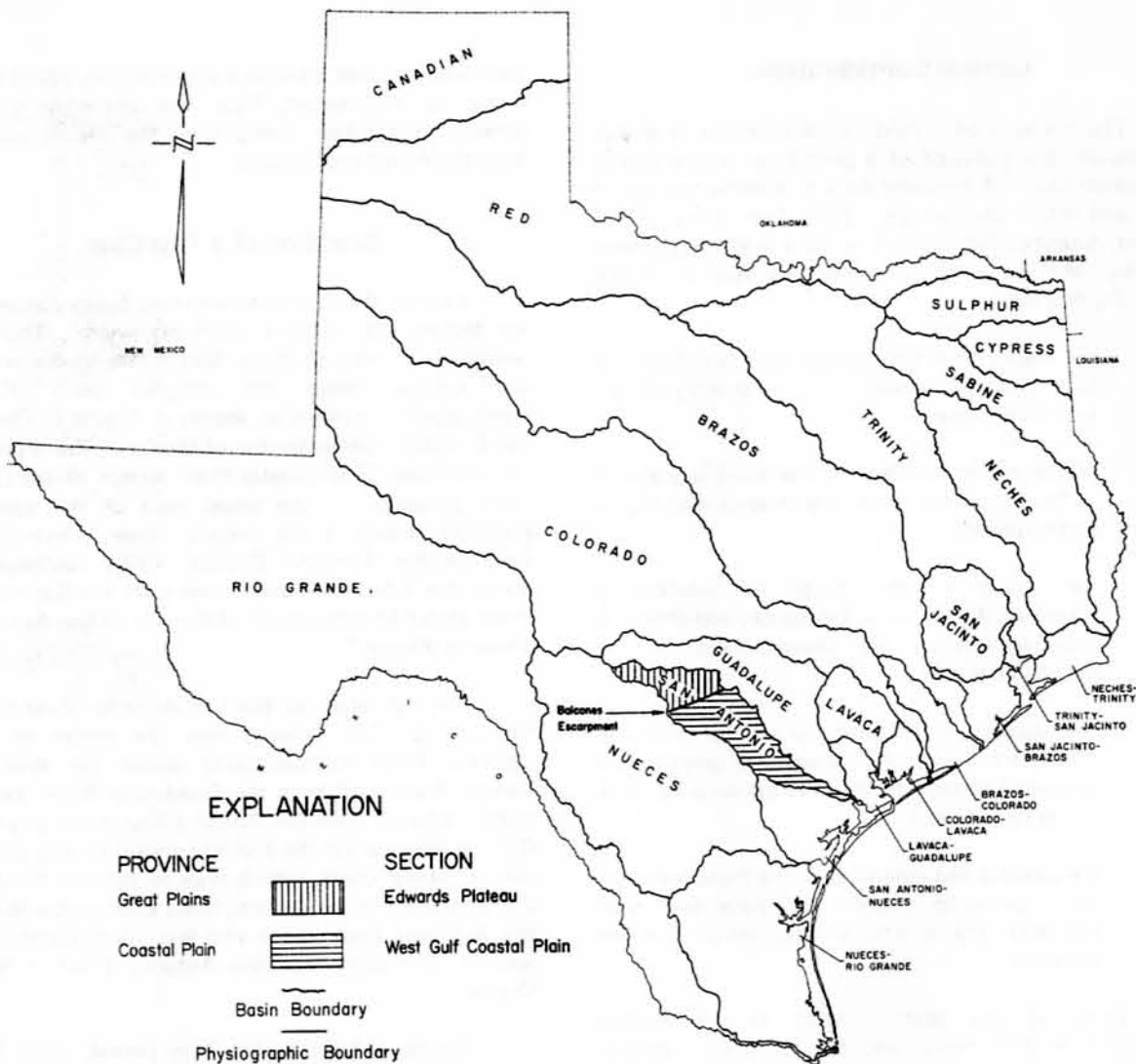
Figure 6.—River Basins in Texas and Physiographic Sections of the San Antonio River Basin

San Antonio River from its source at mile 232* in San Antonio to mile 110 near Runge. A stretch of the Medina River from mile 10 to its confluence with the San Antonio River at mile 210 was also used in order to demonstrate the model's capability to handle converging stream systems. For verification purposes, the stream system was broken down into eight segments as shown in Figure 8, each having its own hydraulic, physical, biological, and chemical characteristics. This particular breakdown was principally dictated by the amount of data that was available for describing the characteristics of the system. The modeling effort was not extended below mile 110 near Runge because the effects of the upstream waste discharges on the BOD, DO, and selected

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* River mileages along the mainstem are measured upstream from the junction of the San Antonio and Guadalupe Rivers, which is designated as mile 0.0. . On tributaries, mileages are measured upstream from the tributary mouth.

conservative minerals in the stream system appeared insignificant at Runge when the June 16-19, 1969 study was conducted.

Input Data and Related Information

Stream Geometry

Values for mean velocity and depth were determined from stream gage rating curves. These values were plotted against discharge as shown in Figure 9 to obtain functional relationships for use in the models.
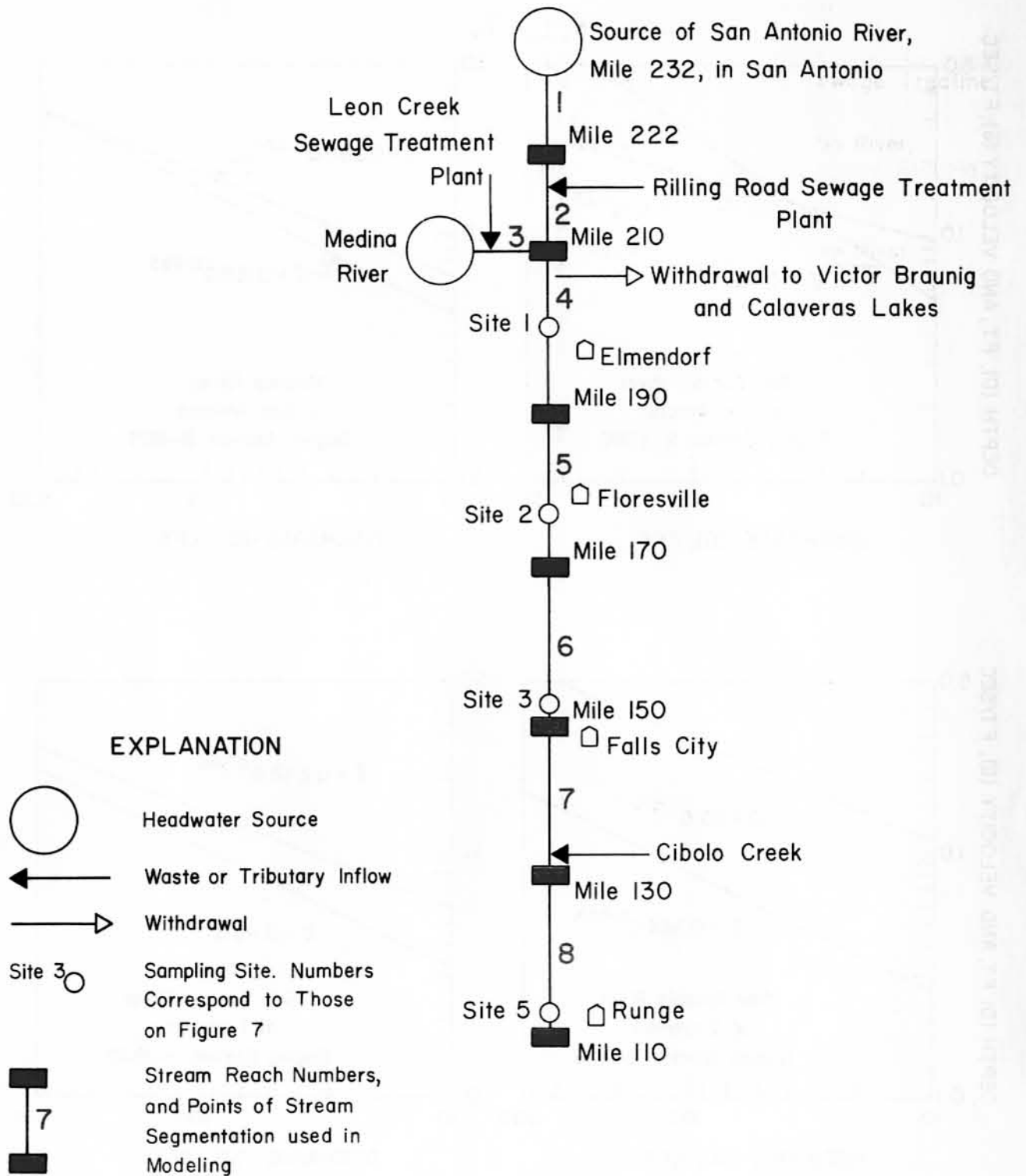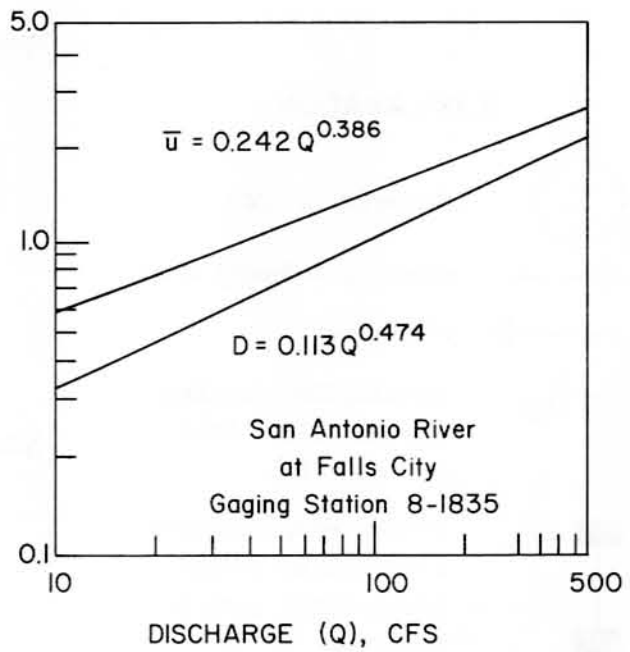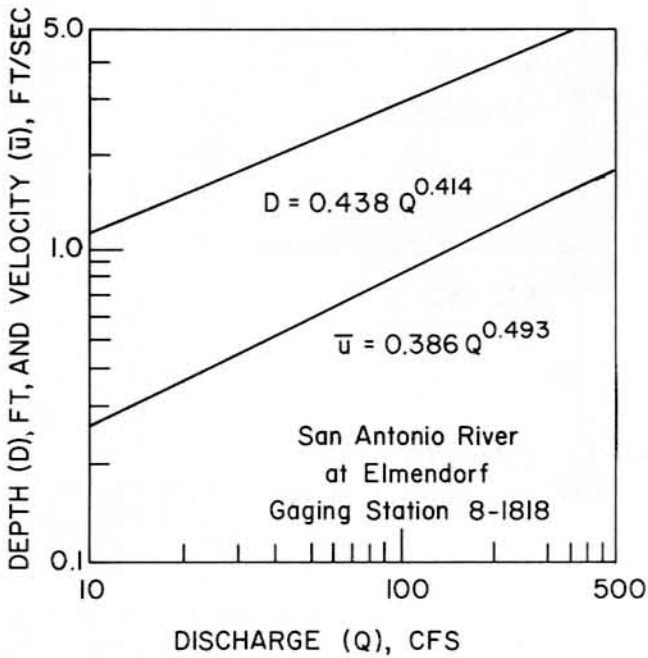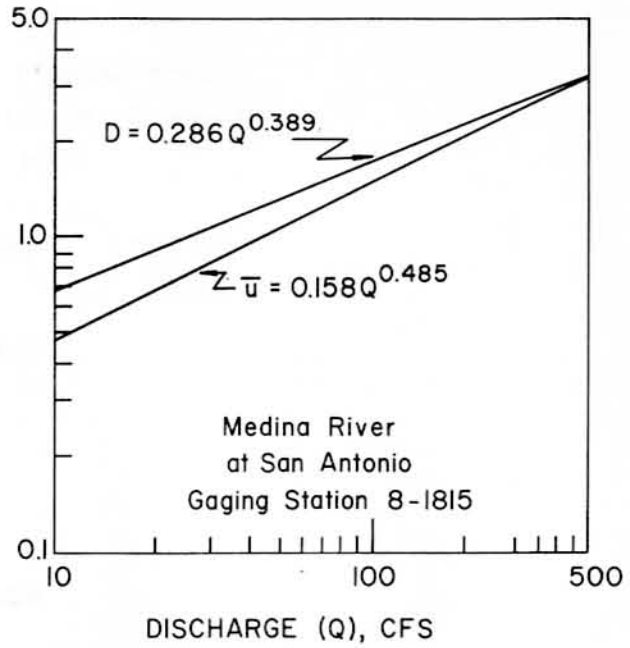
Figure 8
Schematic of Model System
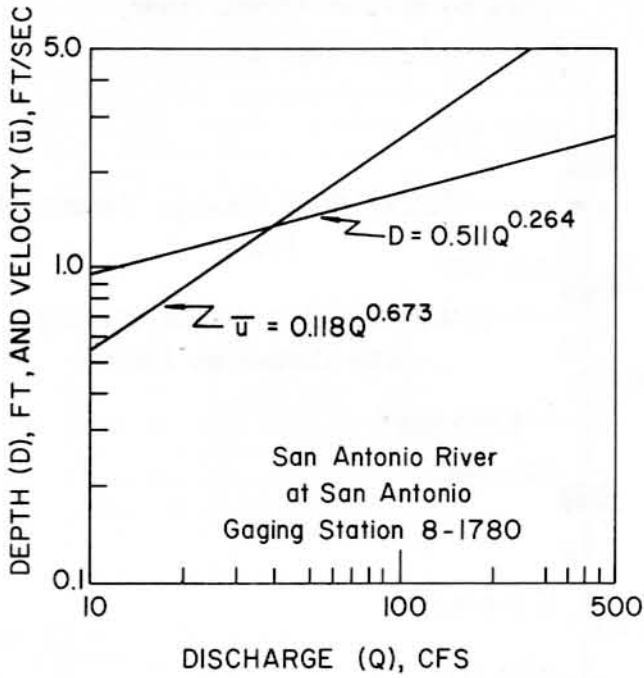
Figure 9
Depth-Velocity-Discharge Relations for
Selected Gaging Stations

Headwater Sources

Headwater flows were determined from stream gaging records for the study period, June 16-19, 1969. Data on the quality of these waters were not readily available. Therefore, reasonable estimates for temperature, BOD and DO, and conservative minerals (total dissolved solids, chloride, and sulfate) were made. These estimates were based on historical data and back calculations using available downstream data.

Waste Loadings and Withdrawals

Waste discharges were determined from flow meters at the plants indicated (Rilling Road Sewage Treatment Plant and Leon Creek Sewage Treatment Plant). The quality of these flows was determined from plant records covering the study period. Estimates of the withdrawals from the San Antonio River to fill Calaveras and Victor Braunig Lakes (powerplant cooling ponds) were obtained from the San Antonio River Authority and the Alamo Area Council of Governments.

Local Climatological Data

Five kinds of climatological-meteorological data were required for simulation of the thermal behavior of the San Antonio River: dry-bulb temperature, wet-bulb temperature, barometric pressure, wind velocity, and cloud cover. Barometric pressure was considered constant over a daily time interval, while the other four quantities were taken as constant over a 3-hour time period.

Reaction Rates K_1 and K_2

The deoxygenation rate constant, K_1, was determined from the U.S. Geological Survey water-quality data and from estimated travel times, Table 4. A graph of BOD_5 versus travel time, T_r, is shown in Figure 10. The reaeration coefficient, K_2, was computed using Equations 52, 53, 57, 58, and 61.

Table 4.—Data for Determination of K_1

RIVER MILE	x (mi)	BOD_5 (mg/l)	Q (cfs)	$\bar{u}$ (fps)	T_r (days)	K_1^{28} (1/day)	K_1^{20} (1/day)
203		14.26					
	13		125.0	0.807	1.02	0.450	0.320
190		9.02					
	20		132.9	.795	2.56	.356	.250
170		5.20					
	20		147.9	.831	4.03	.356	.250
150		3.15					
	20		157.8	1.15	5.09	.290	.204
130		2.24					
	20		183.0	1.23	6.09	.068	.050
110		2.20					

K_1^{28} = deoxygenation rate at 28°C.

K_1^{20} = deoxygenation rate at 20°C.

Data Preparation

The simulation results described in this chapter were obtained by using a constant time interval of integration of 3 hours. The results consider steady-state conditions in the stream. All discharges and quality inputs were considered constant throughout the routing period, except for the climatological-meteorological data.

Operation of the models with a shorter time step and with time variant inputs was not justified in this particular case due to the lack of sufficient data. A summary of the hydrologic-quality data used for verification is given in Table 5.
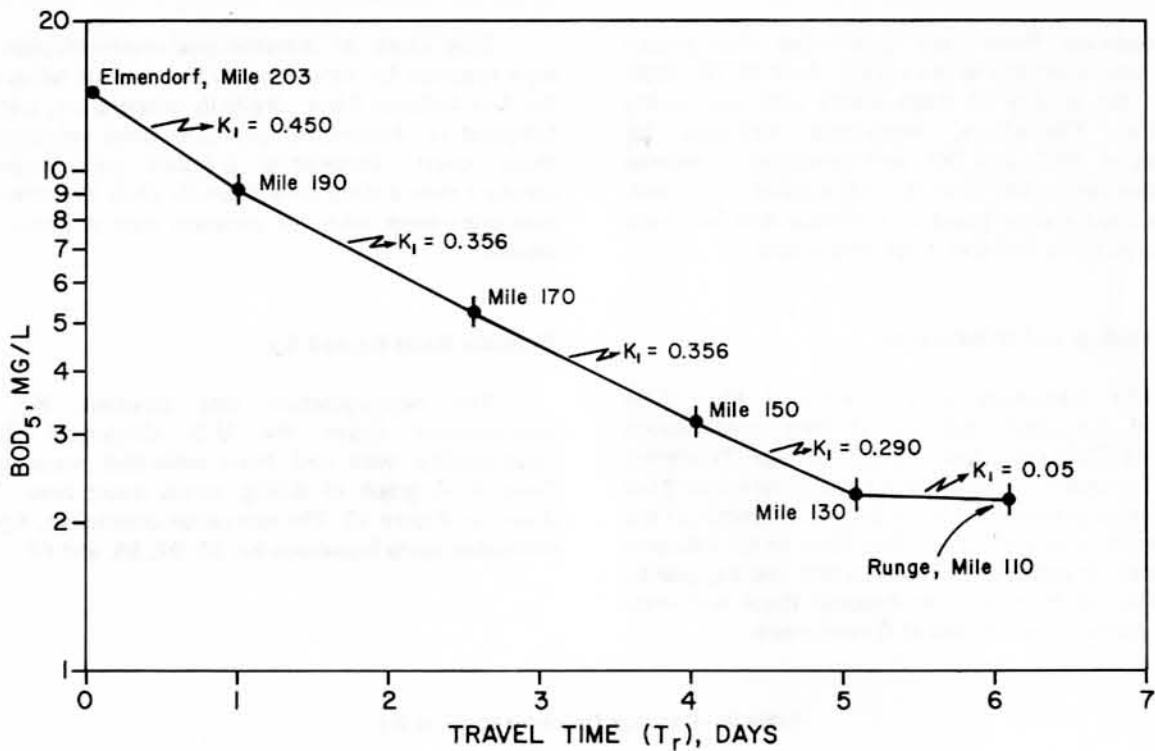
Figure 10.—BOD₅ Versus Travel Time

Figure axes: BOD₅, MG/L (y-axis) versus TRAVEL TIME (T_r), DAYS (x-axis)

Data points: Elmendorf, Mile 203; $K_1 = 0.450$; Mile 190; $K_1 = 0.356$; Mile 170; $K_1 = 0.356$; Mile 150; $K_1 = 0.290$; Mile 130; $K_1 = 0.05$; Runge, Mile 110

Table 5.—Summary of Hydrologic-Quality Data Used
for
Verification of Model

HEADWATER	FLOW (cfs)	TEMP. (°F)	BOD₅ (mg/l)	DO (mg/l)	SULFATE (mg/l)	CHLORIDE (mg/l)	TDS (mg/l)
San Antonio River	14.0	81.4	0.0	7.5	36.0	42.0	299.0
Medina River	60.6	81.4	0.0	7.5	34.4	11.7	234.8
WASTE LOADS OR WITHDRAWALS							
Rilling Road Sewage Treatment Plant	97.0	82.4	34.0	7.0	132.0	139.8	764.9
Leon Creek Sewage Treatment Plant	17.4	78.8	34.0	7.5	113.0	134.0	760.0
Victor Braunig and Calaveras Lakes	64.0	—	—	—	—	—	—
Cibolo Creek	28.0	81.4	1.6	8.0	242.0	169.0	833.0
SAMPLING STATIONS							
Site 1 near Elmendorf	125.0	81.4	14.26	4.97	92.0	91.0	560.0
Site 2 near Floresville	—	81.4	6.02	4.80	105.0	94.0	590.0
Site 3 near Falls City	155.0	81.4	3.05	5.68	136.0	122.0	694.0
Site 5 near Runge	183.0	81.4	2.24	7.20	157.0	152.0	772.0

Simulation Results

Biochemical Oxygen Demand and Dissolved Oxygen

The results from the BOD and DO simulation are shown in Figure 11 along with the observed maximum, mean, and minimum values. Six methods were used for computing the reaeration coefficient. Equation 52 developed by Churchill and others (1962) seemed to give the best agreement, with Langbien's and Durum's (1967) Equation 61 almost as good. However, this should not be interpreted as meaning that Equations 52 and 61 will always give the "best" results. In all probability, there would be cases where the other equations might give the better results. Thus, the selection of the method for determining K_2 must be a subjective decision on the part of the investigator and should be based on a good knowledge of the various characteristics of the prototype stream system. Also, the reaeration coefficient is very sensitive to depth. Therefore, some care must be taken in describing the geometric properties of the stream. Table 6 gives the hydraulic characteristics used in the BOD and DO simulation, along with the computed and estimated K_2's.

As can be seen from Figure 11, the critical (lowest) dissolved-oxygen concentration occurs at approximately river mile 195. This point is approximately 24 miles downstream from the Rilling Road Sewage Treatment Plant which is located at river mile 219 and is 22 miles downstream from the Leon Creek Sewage Treatment Plant located at river mile 7 on the Medina River. The discharge from the Rilling Road Sewage Treatment Plant, about 97.0 cfs, dominates the system to such an extent that the effects of the Leon Creek effluent on the mainstem are almost negligible. The area of the stream near the critical oxygen level, besides being the most sensitive reach of the stream system, was also complicated by the fact that in the same area, approximately 64.0 cfs was being withdrawn from the San Antonio River to supply Calaveras and Victor Braunig Lakes. Both of these impoundments are used as cooling ponds for thermal powerplants. The withdrawal was located at approximately river mile 207.

Conservative Minerals

Three conservative minerals were routed through the system shown in Figure 8. They were sulfate, chloride, and total dissolved solids. As these nonreactive constituents are not affected by any physical, chemical, or biological processes, the routing process is simply a material balance throughout the system. The results from these simulations are shown in Figure 12. The results are very good except in the area of Runge where an external influence (surface runoff, small tributary flows, etc.) apparently was not defined in the available data.

Temperature

There was not enough variation in the San Antonio River system to warrant simulation of the longitudinal distribution of temperature. However, temperature was simulated on a diurnal basis. The results are shown in Figure 13. Computed temperature was found to be very sensitive to changes in stream depth.

Table 6.—Summary of Hydraulic Parameters and Reaction Rates

REACH	AVERAGE FLOW (cfs)	AVERAGE VELOCITY (fps)	AVERAGE DEPTH (ft)	(1) K_2 (1/day)	(2) K_2 (1/day)	(3) K_2 (1/day)	(4) K_2 (1/day)	(5) K_2 (1/day)	(6) K_2 (1/day)	K_1 (1/day)
1	14.0	0.806	1.026	9.0	8.971	11.202	17.934	3.577	6.617	0.320
2	94.8	1.530	1.700	7.5	7.489	7.683	11.642	3.949	4.867	.320
3	72.8	1.311	1.531	7.3	7.477	7.858	11.885	3.798	4.959	.320
4	137.8	.807	3.381	1.5	1.227	1.888	1.995	.828	1.359	.320
5	132.9	.795	3.331	1.0	1.233	1.901	2.011	.829	1.367	.250
6	147.9	.831	3.482	1.0	1.197	1.819	1.908	.825	1.318	.250
7	157.8	1.154	3.577	1.0	1.590	2.060	2.264	1.152	1.499	.204
8	183.0	1.227	3.803	1.0	1.525	1.936	2.104	1.144	1.423	.048

(1) Estimated from field data.
(2) Based on Equation 52 (Churchill and others, 1962).
(3) Based on Equation 53 (O'Conner and Dobbins, 1958).
(4) Based on Equation 56 (Owens, Edwards, and Gibbs, 1964).
(5) Based on Equation 58 (Thackston and Krenkel, 1966).
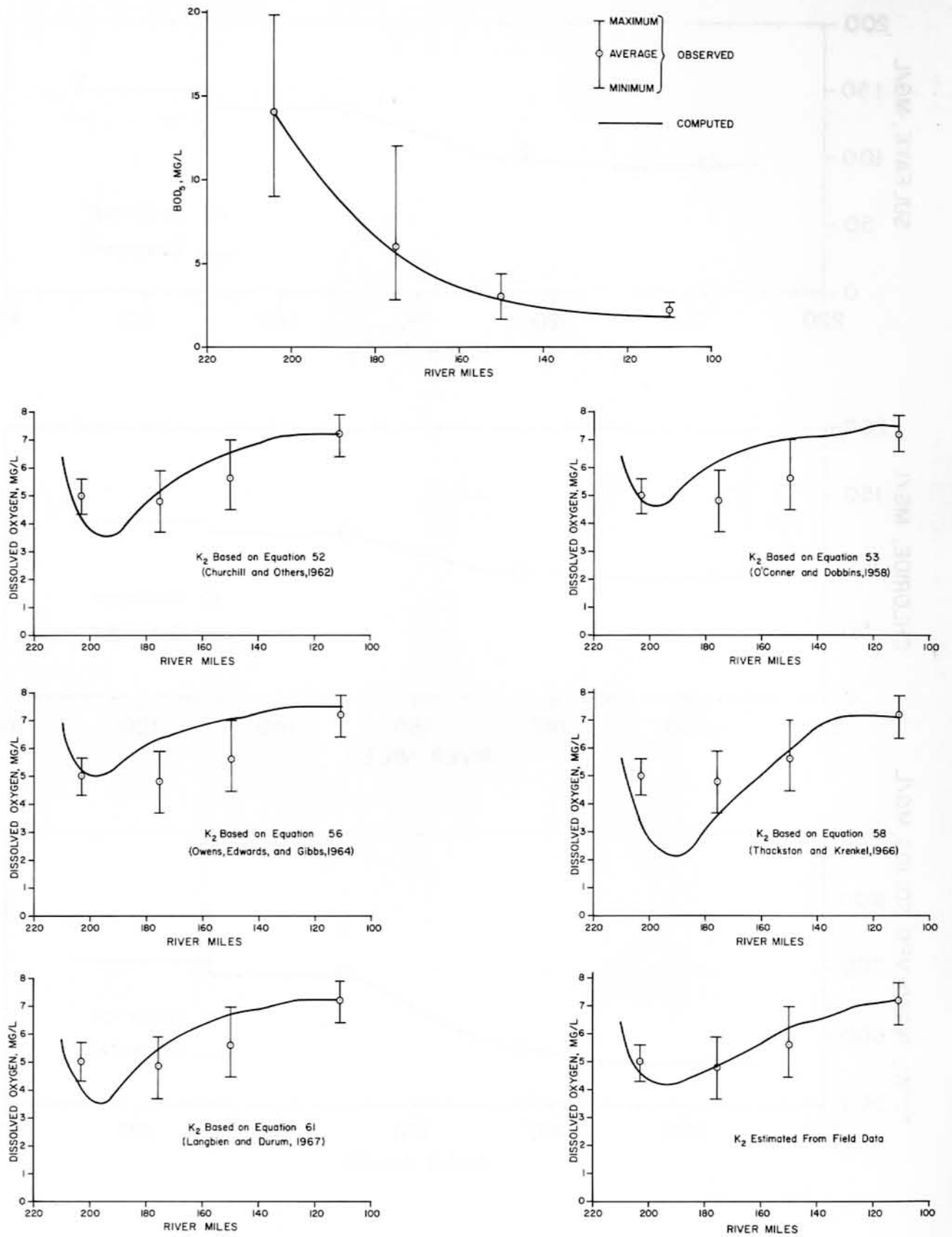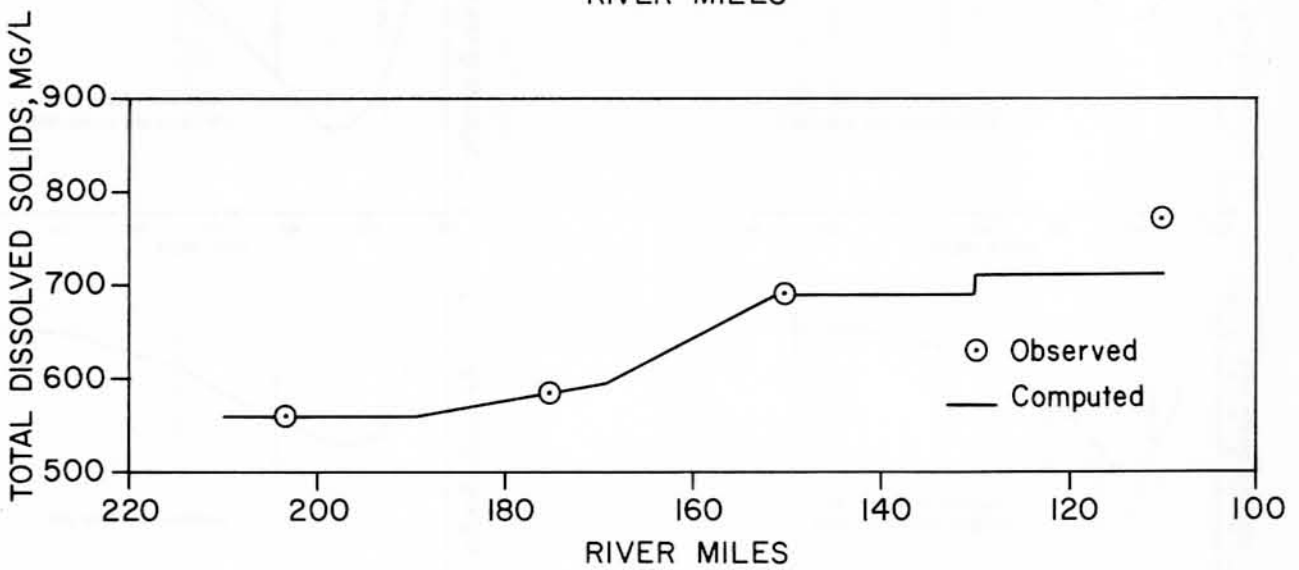(6) Based on Equation 61 (Langbien and Durum, 1967).
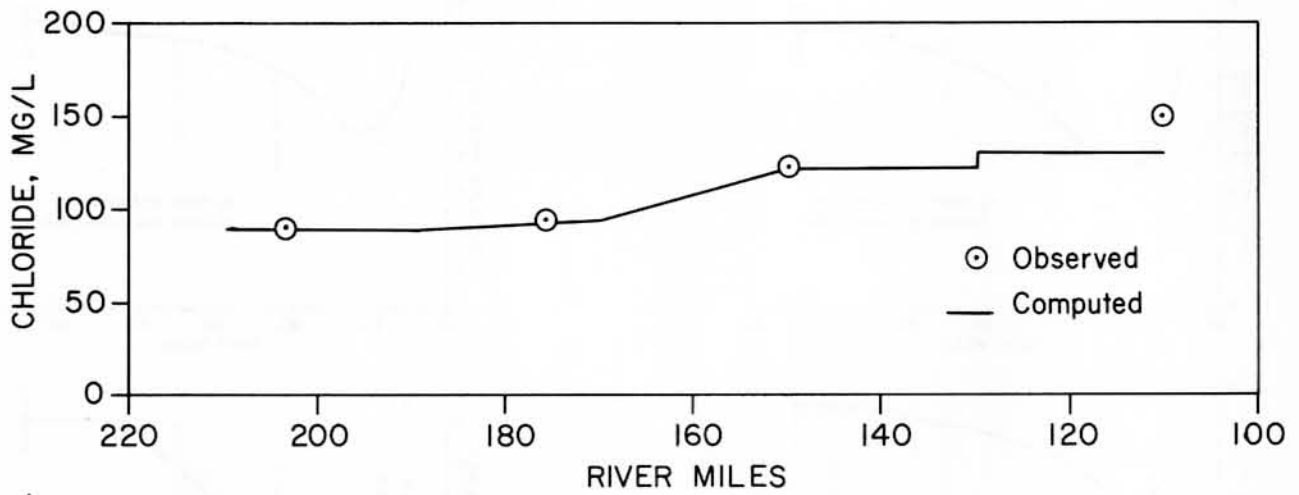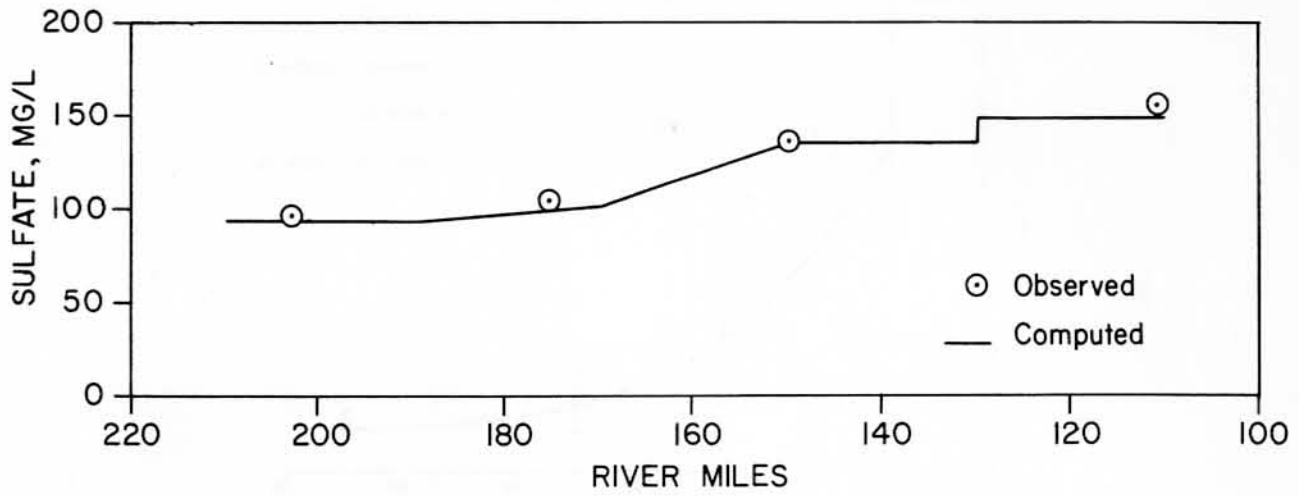
Figure 11
BOD and DO Simulation Results
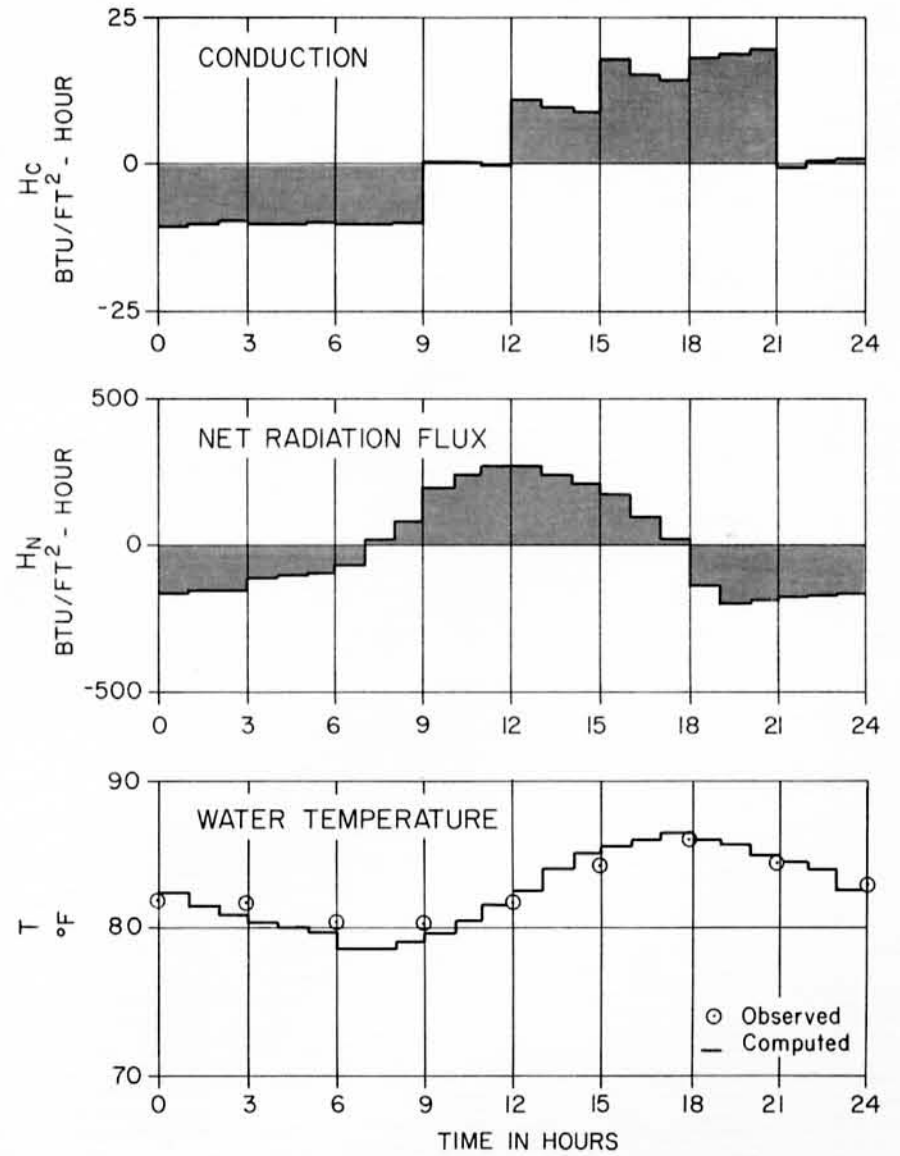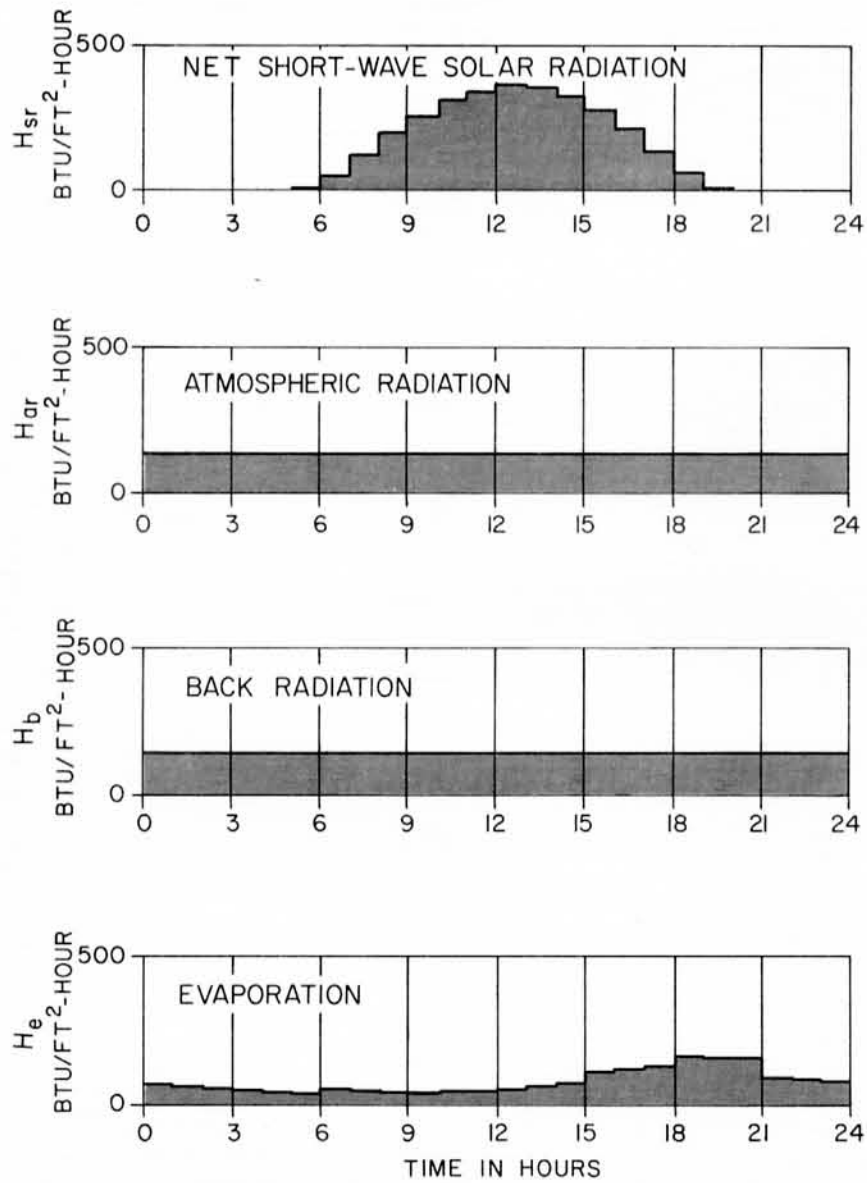
Figure 12

Conservative Minerals Simulation Results

Figure 13

Diurnal Stream Temperature Simulation Results

SUMMARY AND CONCLUSIONS

General

In compliance with the contract dated August 20, 1969, between the Texas Water Development Board and William A. White, Consultant of Frank D. Masch and Associates, QUAL-I, an integrated system of mathematical models, has been developed in a modular form suitable for simulation on a digital computer to route the following parameters through a one-dimensional, fully mixed, branching stream system:

(1)   temperature,

(2)   biochemical oxygen demand and dissolved oxygen, and

(3)   conservative minerals.

Model Capability

QUAL-I was designed to include the following capabilities and characteristics:

(1)   The stream may be discretized into elements of suitable length and variable cross-section (transverse to the channel axis) to obtain any degree of resolution that is warranted.

(2)   QUAL-I can account for heat exchange across the air-water interface and is capable of handling waste inputs and withdrawals at selected points along the channel axis.

(3)   QUAL-I allows for transport by advective and dispersional mechanisms along the principal axis of flow (the longitudinal axis).

(4)   Solutions provide for a temporal and spatial description of temperature, BOD and DO, and conservative mineral variation throughout a stream or canal system.

(5)   It provides for determining flow augmentation requirements based on selected minimum allowable concentrations of dissolved oxygen.

(6)   QUAL-I has an integrated-system capability so that the results of any one sub-model can be used as input to another model provided such feedback is required. Each sub-model also has a "stand-alone" capability. Application of the integrated-system capability was not warranted in this study due to insufficient field data.

(7)   QUAL-I is so structured as to be completely general. It can be applied to any stream or canal system by choosing the appropriate parameters and providing the necessary data which relate to a specific case.

Input Requirements

In addition to the characteristics mentioned above, QUAL-I has been provided with a simplified user-oriented data input format which requires a minimum amount of data manipulation prior to simulation. Physical properties which must be described include the location of waste loadings and withdrawals, the location of stream or canal junctions, and the location and identification of headwater sources available for potential flow augmentation. Input water-quality data include biochemical oxygen demand, dissolved-oxygen concentrations, temperature, and conservative-mineral concentrations. Hydrologic data include headwater flows, waste discharges or withdrawals, tributary inflows, incremental flows (runoff), and depth-velocity-discharge relationships. Reaction rates such as K_1 can be estimated from stream-quality data or laboratory data, while the reaeration coefficient, K_2, can either be estimated or computed by an equation selected from the literature. Mean elevation, latitude, and longitude of the stream basin are required for temperature simulation. Necessary climatological data include cloud cover, dry-bulb temperature, wet-bulb temperature, atmospheric pressure, and wind speed. BOD treatment efficiency can be set at any desired level (percent removal).

Output

The output from QUAL-I yields a time history and spatial description of the distribution of a selected quality constituent throughout the stream or canal system of interest. There are two levels of output. One level is an intermediate summary which allows observation at every point in the system with time. The second level is a final summary of stream conditions after steady-state conditions have been reached, and describes average, maximum, and minimum values of pertinent parameters as well as concentrations of the predicted quality constituents in every reach of the stream or canal system simulated. Several examples of the typical output from QUAL-I are shown in the Appendix.

Future Development and Application

It is appropriate in concluding this report to identify those areas where attention would be most beneficially directed in future utilization of QUAL-I.

First, although the utility of QUAL-I has been sufficiently demonstrated, it is still subject to considerable improvement. Needed improvements can best be determined by continued use of the model. When developing and applying a simulation technique to only one test case, one runs the risk of developing a capability that is case specific. This danger can be eliminated only by additional application of QUAL-I's capabilities, and expanding those capabilities where this is found needed.

Second, there is a general need for more detailed hydrologic and water-quality information in stream systems. Water-quality investigations need to be designed to reduce the uncertainty associated with what is actually happening in the prototype system. A case in point is the difficulty encountered in attempting to distinguish between nitrogenous and carbonaceous BOD's.

Finally, there is the structural code of QUAL-I itself. Certainly, better solution techniques, more efficient data manipulation procedures, and better and more informative means of displaying simulation results will be desired in applying the model to new problems. For the present, however, QUAL-I provides a starting point or foundation from which to approach more effectively one of the more controversial and complicated problems in the management of water resources, the control of water quality in river basins and water transfer systems.

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APPENDIX

EXAMPLES OF TYPICAL OUTPUT

FROM

QUAL-I

CASE A—FLOW AUGMENTATION

Tables A-1, A-2, A-3, A-4, and A-5 illustrate the type of output obtained by employing the flow augmentation capability of QUAL-I.

To test the flow augmentation capability, effluents with a BOD$_5$ concentration of 75 mg/l were modeled as released from the Rilling Road Sewage Treatment Plant at river mile 219 on the San Antonio River and the Leon Creek Sewage Treatment Plant at river mile 7 on the Medina River. Table A-1 shows that these organic loads reduced the steady-state dissolved oxygen concentration to 0.0 from river mile 200 to river mile 190 on the San Antonio River. Both the San Antonio River and the Medina River were considered to be available for flow augmentation.

The amount of flow augmentation required to raise the dissolved oxygen concentration to 4.0 mg/l was estimated and added to the initial headwater flows. The routing computations were then repeated until steady-state conditions were reached throughout the system as shown in Table A-2. Again, dissolved oxygen levels within the system were checked against the prespecified target level of 4.0 mg/l and the flow augmentation required was estimated. This process was repeated until the level of dissolved oxygen throughout the system satisfied the target level of 4.0 mg/l as shown in Table A-5. At this point a summary of the total flow augmentation requirement and a final summary of pertinent information for each reach in the system was written. An example of the final reach summary is shown for Reach 4 in Table A-6.

BIOCHEMICAL OXYGEN DEMAND IN MG/L AFTER   10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1.9	1.9	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4										
2	1.4	1.3	65.0	63.9	62.8	61.8	60.8	59.7	58.7	57.7	56.8	55.8								
3	1.0	1.0	0.9	17.1	16.8	16.5	16.1	15.8	15.5	15.2										
4	38.1	37.0	35.9	34.9	34.0	32.8	31.7	30.6	29.5	28.5	27.6	26.6	25.7	24.8	24.0	23.1	22.3	21.6	20.8	20.1
5	19.5	18.9	18.3	17.7	17.1	16.6	16.1	15.5	15.1	14.6	14.1	13.7	13.3	12.9	12.5	12.1	11.7	11.4	11.0	10.7
6	10.4	10.1	9.8	9.5	9.2	8.9	8.7	8.4	8.2	7.9	7.7	7.5	7.3	7.1	6.9	6.7	6.5	6.3	6.1	6.0
7	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.5	4.4	4.3	4.2	3.8	3.8
8	3.7	3.7	3.7	3.7	3.6	3.6	3.6	3.6	3.6	3.5	3.5	3.5	3.5	3.5	3.4	3.4	3.4	3.4	3.4	3.3

DISSOLVED OXYGEN CONC. IN MG/L AFTER   10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	7.6	7.6	7.7	7.7	7.7	7.7	7.7	7.7	7.7	7.7										
2	7.7	7.7	6.6	6.1	5.7	5.3	5.1	5.0	4.8	4.8	4.7	4.7								
3	7.6	7.6	7.7	7.4	7.3	7.2	7.1	7.1	7.0	7.0										
4	5.3	4.5	3.8	3.1	2.6	2.1	1.7	1.3	1.0	0.8	0.6	0.5	0.4	0.4	0.3	0.4	0.4	0.4	0.5	0.6
5	0.9	1.2	1.4	1.6	1.9	2.1	2.3	2.5	2.6	2.8	3.0	3.1	3.3	3.4	3.6	3.7	3.8	3.9	4.1	4.2
6	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.9	5.0	5.1	5.2	5.3	5.3	5.4	5.5	5.5	5.6	5.6	5.7	5.8
7	5.9	6.0	6.1	6.1	6.2	6.3	6.4	6.4	6.5	6.5	6.6	6.6	6.7	6.7	6.8	6.8	6.8	6.9	7.0	7.0
8	7.1	7.1	7.2	7.2	7.2	7.3	7.3	7.3	7.4	7.4	7.4	7.4	7.4	7.5	7.5	7.5	7.5	7.5	7.5	7.5

* * * REACHES WITH OXYGEN DEFICIT * * *

REACH NO.	REACH IDENTIFICATION	MINIMUM DO.	RIVER MILE
4	RCH #ELMENDORF	0.3	195.0
5	RCH #FLORESVILLE	0.9	189.0

* * * FLOW AUGMENTATION REQUIRED * * *

HEADWATER NO.	HEADWATER IDENTIFICATION	EXISTING HEADWATER FLOW (CFS)	AUG. REQUIRED (CFS)
1	HDW#S.A.R. SOURCE	14.0	67.0
2	HDW#MEDINA RIVER	60.6	67.0

Table A-1.—Flow Augmentation, First Iteration

BIOCHEMICAL OXYGEN DEMAND IN MG/L AFTER   10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7										
2	1.6	1.6	41.1	40.5	39.9	39.4	38.8	38.3	37.7	37.2	36.6	36.1								
3	1.0	1.0	1.0	9.7	9.5	9.4	9.2	9.1	8.9	8.8										
4	23.4	22.8	22.3	21.8	21.3	20.7	20.2	19.7	19.2	18.7	18.2	17.8	17.3	16.9	16.4	16.0	15.6	15.2	14.8	14.4
5	14.1	13.8	13.5	13.2	12.9	12.6	12.3	12.0	11.8	11.5	11.3	11.0	10.8	10.5	10.3	10.1	9.9	9.6	9.4	9.2
6	9.0	8.8	8.6	8.4	8.3	8.1	7.9	7.7	7.6	7.4	7.3	7.1	7.0	6.8	6.7	6.5	6.4	6.2	6.1	6.0
7	5.9	5.8	5.7	5.6	5.6	5.5	5.4	5.3	5.3	5.2	5.1	5.1	5.0	4.9	4.9	4.8	4.7	4.7	4.4	4.3
8	4.3	4.3	4.2	4.2	4.2	4.2	4.2	4.1	4.1	4.1	4.1	4.1	4.1	4.0	4.0	4.0	4.0	4.0	4.0	3.9

DISSOLVED OXYGEN CONC. IN MG/L AFTER   10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	7.5	7.6	7.6	7.6	7.7	7.7	7.7	7.7	7.7	7.7										
2	7.7	7.7	7.0	6.7	6.4	6.2	6.1	6.0	5.9	5.8	5.8	5.8								
3	7.6	7.6	7.6	7.5	7.5	7.4	7.4	7.3	7.3	7.3										
4	6.3	5.8	5.4	5.1	4.8	4.4	4.2	3.9	3.7	3.5	3.3	3.1	3.0	2.9	2.8	2.7	2.6	2.6	2.6	2.5
5	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.1	3.2	3.3	3.4	3.4	3.5	3.6	3.7	3.7	3.8	3.9	4.0	4.0
6	4.1	4.2	4.2	4.3	4.4	4.4	4.5	4.5	4.6	4.7	4.7	4.8	4.8	4.9	5.0	5.0	5.1	5.1	5.2	5.2
7	5.3	5.4	5.5	5.6	5.6	5.7	5.8	5.8	5.9	6.0	6.0	6.1	6.1	6.2	6.2	6.3	6.3	6.3	6.5	6.5
8	6.6	6.7	6.7	6.8	6.8	6.8	6.9	6.9	7.0	7.0	7.0	7.1	7.1	7.1	7.1	7.2	7.2	7.2	7.2	7.3

* * * REACHES WITH OXYGEN DEFICIT * * *

REACH NO.	REACH IDENTIFICATION	MINIMUM DO.	RIVER MILE
4	RCH #ELMENDORF	2.5	190.0
5	RCH #FLORESVILLE	2.6	189.0

* * * FLOW AUGMENTATION REQUIRED * * *

HEADWATER NO.	HEADWATER IDENTIFICATION	EXISTING HEADWATER FLOW (CFS)	AUG. REQUIRED (CFS)
1	HDW#S.A.R. SOURCE	81.0	53.6
2	HDW#MEDINA RIVER	127.6	53.6

Table A-2.—Flow Augmentation, Second Iteration

BIOCHEMICAL OXYGEN DEMAND IN MG/L AFTER   10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.8	1.7	1.7										
2	1.7	1.7	32.0	31.6	31.2	30.8	30.4	30.0	29.6	29.2	28.8	28.4								
3	1.0	1.0	1.0	7.3	7.2	7.1	7.0	6.9	6.8	6.7										
4	18.1	17.7	17.3	17.0	16.6	16.3	15.9	15.5	15.2	14.9	14.5	14.2	13.9	13.6	13.3	13.0	12.7	12.4	12.1	11.9
5	11.6	11.4	11.2	11.0	10.8	10.6	10.4	10.2	10.0	9.8	9.6	9.4	9.3	9.1	8.9	8.7	8.6	8.4	8.3	8.1
6	8.0	7.8	7.7	7.5	7.4	7.2	7.1	7.0	6.8	6.7	6.6	6.5	6.4	6.2	6.1	6.0	5.9	5.8	5.7	5.6
7	5.5	5.4	5.4	5.3	5.3	5.2	5.1	5.1	5.0	5.0	4.9	4.8	4.8	4.7	4.7	4.6	4.6	4.5	4.3	4.2
8	4.2	4.2	4.2	4.2	4.2	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.0	4.0	4.0	4.0	4.0	4.0	3.9	3.9

DISSOLVED OXYGEN CONC. IN MG/L AFTER   10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	7.5	7.6	7.6	7.6	7.6	7.6	7.7	7.7	7.7	7.7										
2	7.7	7.7	7.2	6.9	6.7	6.6	6.5	6.4	6.3	6.3	6.2	6.2								
3	7.5	7.6	7.6	7.5	7.5	7.5	7.4	7.4	7.4	7.4										
4	6.6	6.3	6.0	5.7	5.5	5.3	5.1	4.9	4.7	4.5	4.4	4.2	4.1	4.0	3.9	3.8	3.7	3.7	3.6	3.6
5	3.6	3.6	3.6	3.7	3.7	3.7	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0	4.1	4.1	4.2	4.2	4.3	4.3
6	4.3	4.4	4.4	4.5	4.5	4.6	4.6	4.7	4.7	4.7	4.8	4.8	4.9	4.9	5.0	5.0	5.0	5.1	5.1	5.2
7	5.2	5.3	5.4	5.5	5.5	5.6	5.6	5.7	5.8	5.8	5.9	5.9	6.0	6.0	6.0	6.1	6.1	6.2	6.3	6.3
8	6.4	6.4	6.5	6.5	6.6	6.6	6.7	6.7	6.8	6.8	6.8	6.9	6.9	6.9	7.0	7.0	7.0	7.0	7.1	7.1

* * * REACHES WITH OXYGEN DEFICIT * * *

REACH NO.	REACH IDENTIFICATION	MINIMUM DO.	RIVER MILE
4	RCH #ELMENDORF	3.6	190.0
5	RCH #FLORESVILLE	3.6	189.0

* * * FLOW AUGMENTATION REQUIRED * * *

HEADWATER NO.	HEADWATER IDENTIFICATION	EXISTING HEADWATER FLOW (CFS)	AUG. REQUIRED (CFS)
1	HDW#S.A.R. SOURCE	134.6	25.0
2	HDW#MEDINA RIVER	181.2	25.0

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Table A-3.—Flow Augmentation, Third Iteration

BIOCHEMICAL OXYGEN DEMAND IN MG/L AFTER 10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2.0	1.9	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7										
2	1.7	1.7	29.1	28.7	28.3	28.0	27.6	27.3	26.9	26.6	26.2	25.9								
3	1.0	1.0	1.0	6.6	6.5	6.5	6.4	6.3	6.2	6.1										
4	16.4	16.1	15.8	15.4	15.1	14.8	14.5	14.2	13.9	13.6	13.3	13.0	12.8	12.5	12.2	12.0	11.7	11.5	11.2	11.0
5	10.8	10.6	10.4	10.2	10.0	9.9	9.7	9.5	9.3	9.2	9.0	8.8	8.7	8.5	8.4	8.2	8.1	7.9	7.8	7.7
6	7.5	7.4	7.3	7.1	7.0	6.9	6.8	6.7	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4
7	5.3	5.3	5.2	5.2	5.1	5.0	5.0	4.9	4.9	4.8	4.8	4.7	4.7	4.6	4.6	4.5	4.5	4.4	4.2	4.2
8	4.2	4.1	4.1	4.1	4.1	4.1	4.1	4.1	4.0	4.0	4.0	4.0	4.0	4.0	4.0	3.9	3.9	3.9	3.9	3.9

DISSOLVED OXYGEN CONC. IN MG/L AFTER 10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	7.5	7.6	7.6	7.6	7.6	7.6	7.7	7.7	7.7	7.7										
2	7.7	7.7	7.2	7.0	6.8	6.7	6.6	6.5	6.4	6.4	6.4	6.3								
3	7.5	7.6	7.6	7.5	7.5	7.5	7.5	7.4	7.4	7.4										
4	6.7	6.4	6.2	6.0	5.7	5.5	5.3	5.2	5.0	4.8	4.7	4.6	4.5	4.4	4.3	4.2	4.1	4.0	4.0	3.9
5	3.9	3.9	4.0	4.0	4.0	4.0	4.0	4.1	4.1	4.1	4.2	4.2	4.2	4.2	4.3	4.3	4.3	4.4	4.4	4.5
6	4.5	4.5	4.6	4.6	4.6	4.7	4.7	4.7	4.8	4.8	4.9	4.9	4.9	5.0	5.0	5.0	5.1	5.1	5.1	5.2
7	5.2	5.3	5.4	5.4	5.5	5.6	5.6	5.7	5.7	5.8	5.8	5.9	5.9	6.0	6.0	6.0	6.1	6.1	6.2	6.3
8	6.3	6.4	6.4	6.5	6.5	6.6	6.6	6.7	6.7	6.7	6.8	6.8	6.8	6.9	6.9	6.9	7.0	7.0	7.0	7.0

* * * REACHES WITH OXYGEN DEFICIT * * *

REACH NO.	REACH IDENTIFICATION	MINIMUM DO.	RIVER MILE
4	RCH #ELMENDORF	3.9	190.0
5	RCH #FLORESVILLE	3.9	189.0

* * * FLOW AUGMENTATION REQUIRED * * *

HEADWATER NO.	HEADWATER IDENTIFICATION	EXISTING HEADWATER FLOW (CFS)	AUG. REQUIRED (CFS)
1	HDW#S.A.R. SOURCE	159.6	9.1
2	HDW#MEDINA RIVER	206.2	9.1

Table A-4.—Flow Augmentation, Fourth Iteration

BIOCHEMICAL OXYGEN DEMAND IN MG/L AFTER 10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2.0	1.9	1.9	1.9	1.9	1.8	1.8	1.8	1.8	1.7										
2	1.7	1.7	28.1	27.8	27.4	27.1	26.7	26.4	26.1	25.7	25.4	25.1								
3	1.0	1.0	1.0	6.4	6.3	6.2	6.2	6.1	6.0	5.9										
4	15.9	15.6	15.3	15.0	14.7	14.4	14.1	13.8	13.5	13.2	12.9	12.7	12.4	12.1	11.9	11.6	11.4	11.2	10.9	10.7
5	10.5	10.3	10.2	10.0	9.8	9.6	9.5	9.3	9.1	9.0	8.8	8.7	8.5	8.4	8.2	8.1	7.9	7.8	7.7	7.5
6	7.4	7.3	7.1	7.0	6.9	6.8	6.7	6.6	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3
7	5.3	5.2	5.2	5.1	5.0	5.0	4.9	4.9	4.8	4.8	4.7	4.7	4.6	4.6	4.5	4.5	4.4	4.4	4.2	4.1
8	4.1	4.1	4.1	4.1	4.1	4.1	4.0	4.0	4.0	4.0	4.0	4.0	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9

DISSOLVED OXYGEN CONC. IN MG/L AFTER 10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	7.5	7.6	7.6	7.6	7.6	7.6	7.7	7.7	7.7	7.7										
2	7.7	7.7	7.2	7.0	6.9	6.7	6.6	6.6	6.5	6.4	6.4	6.4								
3	7.5	7.6	7.6	7.5	7.5	7.5	7.5	7.4	7.4	7.4										
4	6.7	6.5	6.2	6.0	5.8	5.6	5.4	5.3	5.1	4.9	4.8	4.7	4.6	4.5	4.4	4.3	4.2	4.2	4.1	4.0
5	4.1	4.1	4.6	4.6	4.1	4.1	4.1	4.2	4.2	4.2	4.2	4.3	4.3	4.3	4.3	4.4	4.4	4.4	4.5	4.5
6	4.5	4.6	4.6	4.6	4.7	4.7	4.7	4.8	4.8	4.8	4.9	4.9	5.0	5.0	5.0	5.1	5.1	5.1	5.2	5.2
7	5.3	5.3	5.4	5.4	5.5	5.6	5.6	5.7	5.7	5.8	5.8	5.9	5.9	5.9	6.0	6.0	6.1	6.1	6.2	6.3
8	6.3	6.4	6.4	6.5	6.5	6.6	6.6	6.6	6.7	6.7	6.8	6.8	6.8	6.9	6.9	6.9	6.9	7.0	7.0	7.0

TOTAL FLOW AUGMENTATION REQUIRED

HEADWATER NO.	HEADWATER IDENTIFICATION	INITIAL HEADWATER FLOW (CFS)	AUG. REQUIRED (CFS)
1	HDW#S.A.R. SOURCE	14.0	154.8
2	HDW#MEDINA RIVER	50.6	154.8

Table A-5.—Flow Augmentation, Final Iteration

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REACH NO.   1 (RCH #S A R SOURCE   )

PARAMETER		HEAD OF REACH	END OF REACH	MAXIMUM	MINIMUM	AVERAGE
RIVER MILE	■	232.0	222.0			
FLOW (CFS)	■	168.780	168.780	168.780	168.780	168.779
VELOCITY (FPS)	■	1.856	1.856	1.856	1.856	1.856
DEPTH (FT)	■	1.979	1.979	1.979	1.979	1.979
K1 (1/DAY) ! 20 C	■	0.320	0.320	0.320	0.320	0.320
K2 (1/DAY) ! 20 C	■	6.748	6.748	6.748	6.748	6.748
K3 (1/DAY) ! 20 C	■	0.000	0.000	0.000	0.000	0.000
TEMP (F)	■	81.400	81.400	81.400	81.400	81.400
BOD (MG/L)	■	1.971	1.726	1.971	1.726	1.846
DO (MG/L)	■	7.537	7.673	7.673	7.537	7.622
CONS ONE (MG/L)	■	0.000	0.000	0.000	0.000	0.000
CONS TWO (MG/L)	■	0.000	0.000	0.000	0.000	0.000
CONS THREE (MG/L)	■	0.000	0.000	0.000	0.000	0.000

Table A-6.—Flow Augmentation, Final Summary for Reach 4

Table B-1 illustrates the type of output obtained by exercising the temperature routing option of QUAL-I.

To test QUAL-I's capability to simulate the thermal behavior of a stream, a withdrawal of 64.0 cfs was arbitrarily taken out of the system at river mile 205.0 and put back into the system at river mile 204.0 after arbitrarily giving it a temperature of 115.0°F. This process is a reasonable characterization of the discharge of cooling waters from a typical thermal powerplant. Table B-1 represents the diurnal behavior throughout the system after steady-state conditions have been reached. An example of the final reach summary is shown for Reach 5 in Table 2.

TEMPERATURE IN DEGREES FARENHEIT AFTER 9.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	80.3	79.7	79.5	79.5	79.7	79.9	80.2	80.4	80.7	80.9										
2	81.0	81.1	82.0	81.7	81.6	81.5	81.5	81.6	81.8	82.0	82.2	82.5								
3	78.4	78.2	78.2	78.4	78.6	78.9	79.3	79.7	80.2	80.7										
4	82.5	83.6	84.5	85.2	84.9	94.8	94.4	94.0	93.6	93.3	92.9	92.6	92.3	92.0	91.8	91.5	91.3	91.2	91.0	90.8
5	90.6	90.5	90.3	90.1	90.0	89.8	89.7	89.6	89.4	89.3	89.2	89.1	89.0	88.9	88.8	88.8	88.7	88.6	88.5	88.5
6	88.4	88.4	88.3	88.2	88.2	88.1	88.1	88.1	88.0	88.0	87.9	87.9	87.9	87.8	87.8	87.8	87.8	87.7	87.7	87.7
7	87.6	87.6	87.6	87.5	87.5	87.5	87.5	87.4	87.4	87.4	87.4	87.3	87.3	87.3	87.3	87.3	87.2	87.2	86.6	86.6
8	86.6	86.6	86.6	86.6	86.6	86.6	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.8

TEMPERATURE IN DEGREES FARENHEIT AFTER 9.25 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	79.1	77.3	75.9	74.9	74.2	73.8	73.5	73.3	73.2	73.2										
2	73.2	73.2	80.7	80.1	79.5	79.0	78.6	78.2	78.0	77.7	77.5	77.4								
3	77.8	77.0	76.2	76.3	75.8	75.5	75.2	75.1	75.0	75.0										
4	76.8	77.5	78.4	79.2	78.9	90.7	90.5	90.2	89.9	89.7	89.4	89.1	88.8	88.6	88.3	88.1	87.9	87.6	87.5	87.3
5	87.1	86.9	86.8	86.6	86.5	86.3	86.2	86.1	86.0	85.8	85.7	85.7	85.6	85.5	85.4	85.3	85.2	85.2	85.1	85.1
6	85.0	84.9	84.9	84.8	84.8	84.8	84.7	84.7	84.7	84.6	84.6	84.6	84.5	84.5	84.5	84.5	84.4	84.4	84.4	84.4
7	84.3	84.3	84.3	84.3	84.3	84.2	84.2	84.2	84.2	84.2	84.2	84.1	84.1	84.1	84.1	84.1	84.1	84.1	83.8	83.7
8	83.7	83.7	83.7	83.7	83.7	83.7	83.7	83.7	83.7	83.7	83.7	83.7	83.7	83.7	83.8	83.8	83.8	83.8	83.8	83.8

TEMPERATURE IN DEGREES FARENHEIT AFTER 9.50 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	86.4	89.4	91.1	91.9	92.2	92.1	92.0	91.8	91.6	91.4										
2	91.2	91.1	84.8	85.8	86.6	87.2	87.6	87.8	88.0	88.0	88.0	88.0								
3	81.6	83.7	85.1	84.8	85.5	86.0	86.3	86.5	86.6	86.6										
4	86.3	84.9	84.1	83.7	84.8	94.9	94.7	94.5	94.3	94.1	93.9	93.6	93.4	93.1	92.9	92.7	92.4	92.2	92.0	91.8
5	91.6	91.3	91.1	91.0	90.8	90.6	90.5	90.3	90.2	90.0	89.9	89.8	89.7	89.6	89.5	89.4	89.3	89.2	89.1	89.1
6	89.0	88.9	88.9	88.8	88.7	88.7	88.6	88.6	88.5	88.5	88.5	88.4	88.4	88.3	88.3	88.3	88.3	88.2	88.2	88.2
7	88.1	88.1	88.1	88.0	88.0	88.0	87.9	87.9	87.9	87.9	87.8	87.8	87.8	87.8	87.7	87.7	87.7	87.7	87.1	87.1
8	87.1	87.1	87.1	87.1	87.1	87.1	87.1	87.1	87.1	87.0	87.0	87.0	87.0	87.0	87.0	87.0	87.0	87.0	87.0	87.0

Table B-1. Diurnal Temperature Simulation, Intermediate Summary

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	84.1	86.4	88.1	89.4	90.3	91.0	91.4	91.7	91.9	92.0										
2	92.1	92.1	84.2	85.1	86.0	86.8	87.6	88.3	88.9	89.4	89.9	90.2								
3	80.6	82.4	84.1	84.2	85.5	86.6	87.5	88.3	89.0	89.6										
4	90.3	90.3	90.0	89.5	89.5	97.4	96.9	96.4	96.0	95.7	95.4	95.2	95.0	94.8	94.6	94.4	94.2	94.1	93.9	93.7
5	93.5	93.3	93.2	93.0	92.8	92.7	92.5	92.4	92.3	92.2	92.0	91.9	91.8	91.7	91.6	91.6	91.5	91.4	91.3	91.3
6	91.2	91.1	91.1	91.0	91.0	90.9	90.9	90.8	90.8	90.7	90.7	90.7	90.6	90.6	90.6	90.5	90.5	90.5	90.4	90.4
7	90.4	90.3	90.3	90.3	90.2	90.2	90.2	90.1	90.1	90.1	90.0	90.0	90.0	90.0	89.9	89.9	89.9	89.9	89.0	89.1
8	89.1	89.1	89.1	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	80.3	79.7	79.5	79.5	79.7	79.9	80.2	80.5	80.7	80.9										
2	81.0	81.1	82.0	81.7	81.6	81.5	81.5	81.6	81.8	82.0	82.2	82.5								
3	78.4	78.2	78.2	78.4	78.6	78.9	79.3	79.7	80.2	80.7										
4	82.5	83.6	84.5	85.2	84.9	94.8	94.4	94.0	93.6	93.3	92.9	92.6	92.3	92.0	91.8	91.5	91.3	91.2	91.0	90.8
5	90.6	90.5	90.3	90.1	90.0	89.8	89.7	89.6	89.4	89.3	89.2	89.1	89.0	88.9	88.8	88.8	88.7	88.6	88.5	88.5
6	88.4	88.4	88.3	88.3	88.2	88.2	88.1	88.1	88.0	88.0	88.0	87.9	87.9	87.9	87.8	87.8	87.8	87.8	87.7	87.7
7	87.7	87.6	87.6	87.6	87.6	87.5	87.5	87.5	87.4	87.4	87.4	87.4	87.4	87.3	87.3	87.3	87.3	87.3	86.6	86.6
8	86.6	86.6	86.6	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.8	86.8	86.8	86.8	86.8	86.8	86.8	86.8	86.8	86.8

Table B-1. Diurnal Temperature Simulation, Intermediate Summary--Continued

REACH NO.   5 (FLORESVILLE AREA   )

PARAMETER		HEAD OF REACH	END OF REACH	MAXIMUM	MINIMUM	AVERAGE
RIVER MILE	=	190.0	170.0			
FLOW (CFS)	=	189.750	204.000	204.000	189.750	196.875
VELOCITY (FPS)	=	.921	.949	.949	.921	.935
DEPTH (FT)	=	3.860	3.978	3.978	3.860	3.920
K1 (1/DAY)	=	.250	.250	.250	.250	.250
K2 (1/DAY)	=	0.000	0.000	0.000	0.000	0.000
K3 (1/DAY)	=	0.000	0.000	0.000	0.000	0.000
TEMP (F)	=	90.639	88.476	90.639	88.476	89.375
BOD (MG/L)	=	1.000	1.000	1.000	1.000	1.000
DO (MG/L)	=	6.000	6.000	6.000	6.000	6.000
CONS ONE (MG/L)	=	0.000	0.000	0.000	0.000	0.000
CONS TWO (MG/L)	=	0.000	0.000	0.000	0.000	0.000
CONS THREE (MG/L)	=	0.000	0.000	0.000	0.000	0.000

Table B-2.   Diurnal Temperature Simulation, Final Summary for Reach 5

CASE C—CONSERVATIVE MINERAL SIMULATION

Tables C-1 and C-2 illustrate the type of output obtained by applying QUAL-I's capability to route concentrations of conservative minerals.

Using available field data, three conservative minerals (sulfate, chloride, and total dissolved solids) were routed until steady-state conditions were reached. Table C-1 illustrates the intermediate summary, which describes the conservative-mineral concentrations at every river mile in the stream system. An example of the final reach summary is shown for Reach 2 in Table C-2.

CONSERVATIVE MINERAL   I (MG/L X 10) = SULFATE AFTER   10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6										
2	3.6	3.6	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0								
3	3.4	3.4	3.4	5.2	5.2	5.2	5.2	5.2	5.2	5.2										
4	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2
5	9.2	9.3	9.3	9.4	9.5	9.5	9.6	9.6	9.7	9.7	9.8	9.8	9.9	9.9	10.0	10.0	10.0	10.1	10.1	10.2
6	10.4	10.6	10.7	10.9	11.1	11.3	11.5	11.6	11.8	12.0	12.1	12.3	12.5	12.6	12.8	13.0	13.1	13.3	13.4	13.6
7	13.6	13.7	13.7	13.8	13.8	13.9	13.9	14.0	14.0	14.0	14.1	14.1	14.2	14.2	14.3	14.3	14.3	14.4	15.7	15.8
8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.9	15.9	15.9	15.9

CONSERVATIVE MINERAL   II (MG/L X 10) = CHLORIDE AFTER   10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2										
2	4.2	4.2	12.7	12.7	12.7	12.7	12.7	12.7	12.7	12.7	12.7	12.7								
3	1.2	1.2	1.2	3.9	3.9	3.9	3.9	3.9	3.9	3.9										
4	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1
5	9.1	9.1	9.1	9.2	9.2	9.2	9.2	9.2	9.2	9.3	9.3	9.3	9.3	9.3	9.3	9.3	9.4	9.4	9.4	9.4
6	9.5	9.7	9.8	10.0	10.1	10.3	10.4	10.6	10.7	10.9	11.0	11.1	11.3	11.4	11.5	11.7	11.8	11.9	12.1	12.2
7	12.3	12.3	12.4	12.4	12.5	12.5	12.6	12.6	12.7	12.8	12.8	12.9	12.9	13.0	13.0	13.1	13.1	13.1	13.7	13.7
8	13.8	13.8	13.8	13.8	13.8	13.8	13.9	13.9	13.9	13.9	13.9	13.9	13.9	14.0	14.0	14.0	14.0	14.0	14.0	14.0

CONSERVATIVE MINERAL III (MG/L X 10) = T. D. S. AFTER   10.00 DAYS

RCH/CL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	29.9	29.9	29.9	29.9	29.9	29.9	29.9	29.9	29.9	29.9										
2	29.9	29.9	70.6	70.6	70.6	70.6	70.6	70.6	70.6	70.6	70.6	70.6								
3	23.5	23.5	23.5	35.2	35.2	35.2	35.2	35.2	35.2	35.2										
4	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0
5	56.1	56.3	56.5	56.6	56.8	56.9	57.1	57.3	57.4	57.6	57.7	57.9	58.0	58.1	58.3	58.4	58.6	58.7	58.8	59.0
6	59.6	60.1	60.7	61.2	61.8	62.3	62.9	63.4	63.9	64.4	65.0	65.5	66.0	66.5	67.0	67.5	67.9	68.4	68.9	69.4
7	69.6	69.8	70.1	70.3	70.5	70.7	71.0	71.2	71.4	71.6	71.8	72.0	72.2	72.4	72.6	72.7	72.9	73.1	74.6	74.8
8	74.8	74.8	74.9	74.9	74.9	74.9	75.0	75.0	75.0	75.0	75.0	75.1	75.1	75.1	75.1	75.1	75.1	75.1	75.1	75.2

Table C-1. Conservative Mineral Simulation, Intermediate Summary

REACH NO.   2 (RCH = RILLING ROAD)

PARAMETER		HEAD OF REACH	END OF REACH	MAXIMUM	MINIMUM	AVERAGE
RIVER MILE	=	222.0	210.0			
FLOW (CFS)	=	14.000	111.000	111.000	14.000	94.833
VELOCITY (FPS)	=	.806	1.613	1.613	.806	1.530
DEPTH (FT)	=	1.026	1.772	1.772	1.026	1.700
K1 (1/DAY)	=	.320	.320	.320	.320	.320
K2 (1/DAY)	=	0.000	0.000	0.000	0.000	0.000
K3 (1/DAY)	=	0.000	0.000	0.000	0.000	0.000
TEMP (F)	=	81.400	81.400	81.400	81.400	81.400
BOD (MG/L)	=	1.000	1.000	1.000	1.000	1.000
DO (MG/L)	=	6.000	6.000	6.000	6.000	6.000
CONS ONE (MG/L)	=	36.000	119.883	119.889	36.000	105.908
CONS TWO (MG/L)	=	42.000	127.454	127.462	42.000	113.219
CONS THREE (MG/L)	=	299.000	706.091	706.126	299.000	638.274

Table C-2.   Conservative Mineral Simulation, Final Summary for Reach 2