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ANALYSIS OF UNIT HYDROGRAPHS FOR SMALL WATERSHEDS IN TEXAS

By

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PREFACE

There is a scarcity of data concerning the analysis of flood hydrographs for small watersheds in Texas. Such data are needed as a guide in the design of structures on streams draining small watersheds. When adequate data are not available, the designer must use empirical methods to estimate the shape of the runoff hydrograph.

The purpose of this study is to analyze surface-runoff hydrographs for three small watersheds in Texas, develop characteristic dimensionless and unit hydrograph shapes, and compare these shapes with empirical ones. Results of studies of this type should be of assistance to designers of small watershed projects.

This report is a reproduction of a thesis submitted in August of 1964 to the Graduate School of The University of Texas in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

The writer wishes to express his sincere appreciation to all who helped to make this study possible. Those to whom special acknowledgment is due are the following:

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The Computation Center of The University of Texas who provided the facilities for making the necessary computations.

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ANALYSIS OF UNIT HYDROGRAPHS FOR SMALL WATERSHEDS IN TEXAS

PURPOSE

The purpose of this investigation was to study the effectiveness of using the S-curve hydrograph as an aid in estimating the rainfall excess durations for isolated storms and to develop the unit hydrographs for these storms. Runoff characteristics of three selected watersheds were studied. Computer programs were developed to provide a means for rapid analysis of rainfall and runoff data from isolated storms. Characteristic dimensionless and unit hydrograph shapes were determined for each of the watersheds studied. The dimensionless unit hydrographs were compared with plots representing several widely-used empirical curves.

PREVIOUS INVESTIGATIONS

Unit Hydrograph Investigations

Since the first structure was placed on a stream, engineers have sought methods for predicting the characteristics of the complex phenomena known as flood runoff. It was quite common for rainfall and runoff records of floods to be studied to develop relationships by which design predictions might be made. It is important that these predictions be accurate. If the estimated flood magnitude is too large, money will be needlessly wasted on an oversized structure. If the prediction is too small, the structure may be destroyed. To make these estimates, procedures were developed for analyzing the available data and then synthesizing the results of the analysis into design predictions.

Procedures for the analysis of flood runoff data were developed first. Foremost among the early investigators was Sherman (1932) who suggested the concept of the unit hydrograph. The unit graph as proposed by Sherman was one "representing 1 inch of runoff from a 24-hour rainfall." Sherman also showed that the hydrograph of runoff for a continuous uniform rainfall would result in an S-shaped curve. The procedure for computing a hydrograph, given a unit hydrograph, was discussed in detail.

After Sherman suggested the unit graph method, Hoyt and others (1936) discussed its characteristics and developed unit graphs for selected drainage basins within the United States. A unit hydrograph was defined by Hoyt to be "a hydrograph of surface runoff resulting from rainfall within a unit of time, as a day or an hour."

Since derivation of a unit hydrograph presupposes the existence of a flood hydrograph from a gaged area, the unit graph procedure was limited in usefulness only to those areas from which runoff was measured. To extend the utility of this method, early researchers such as Snyder and Commons developed means for predicting flood hydrographs for ungaged areas.

Snyder (1938) presented a method that, as he stated, was "mostly empirical" for deriving unit hydrographs for ungaged areas. Snyder defined a unit graph as the hydrograph for 1 inch of surface runoff from a given area for a specified duration and areal distribution of rainfall. The data on which his study was based were taken in the Appalachian Highlands from drainage areas that ranged in size from 10 to 10,000 square miles. The method provided a means of predicting the time base, peak discharge, and "lag" time for a particular basin. The "lag" time as defined by Snyder was the time from the centroid of the runoff-producing rainfall to the peak of the unit hydrograph. Three equations were developed by Snyder for predicting the lag time, peak discharge, and time base of a unit hydrograph from measureable basin characteristics.

The lag time, t_p, was found to be a function of the length of the main stream, L, and the length of the main stream to the centroid of the drainage basin, L_{ca}, as shown in equation 1. The lag time was expressed in hours.

$$t_p = C_t \ (L \ L_{ca})^{0.3} \tag{1}$$

The coefficient, C_t, was found to vary in the Appalachian Highlands from 1.8 to 2.2. The peak discharge, q_p, and time base, T, of the unit hydrograph were expressed in cfs (cubic feet per second) per square mile and days respectively. Both were found to be functions of the lag time as shown in equations 2 and 3.

$$q_p = \frac{C_p(640)}{t_p} \tag{2}$$

$$T = 3 + 3(t_p/24) \tag{3}$$

The coefficient, C_p, was found to vary from 0.56 to 0.69. In order to develop a unit hydrograph using Snyder's method, it is necessary to evaluate C_t and C_p values from known floods in the area and to sketch a hydrograph--under which area equals 1 inch of runoff--through the origin, peak, and end point that have been empirically determined using the equations listed above. Snyder noted under his section "Limitations and Conclusions" that "The equations and coefficients given are based mainly on the fairly mountainous Appalachian Highlands and may need considerable adjustment to take care of streams in the relatively flat Middle West."

Snyder's method has been modified, and is used frequently by the U. S. Corps of Engineers to predict flood runoff in Texas. It is interesting to note how the two Snyder coefficients vary in Texas floods. Based on data determined by the U. S. Army District, Fort Worth, Texas, the values of C_t and C_p have been found to vary from 0.6 to 6.0 and 0.48 to 0.94, respectively. The coefficient, C_t, is a measure of the differences in slope and channel storage

between drainage basins. The coefficient, C_p, represents the effects of such factors as channel storage on the flood wave. A tabulation of selected Snyder coefficients determined by the U. S. Army District, Fort Worth, Texas, is shown in Table 1.

An interesting item concerning the original Snyder method was that it was impossible, using the equation for the time base of the unit hydrograph, to compute a value for time base that was less than 3 days. Snyder noted that this was unusual, but stated, "It appears that the additional time required must be due to ground storage or delay." The equation for the computation of time base is not included in the recent U. S. Corps of Engineers design manual (1959), which describes the Snyder method. A substitute method of evaluating other points on the empirical unit hydrograph is described. Using this method the width of the unit graph can be determined at ordinates of 50 and 75 percent of the peak discharge.

Another method that is frequently used in Texas is the method proposed by Commons (1942). Commons developed a dimensionless hydrograph by trial that was considered to be an average condition for the single peak hydrographs on which he had data. Commons noted that "The study of major floods has disclosed what appeared to be a regular pattern of distribution of the flow." Morgan and Johnson (1962) stated that Commons may have envisioned that he had discovered a sort of normal distribution for floods. One need only know the total volume of runoff in inches, the peak discharge, and the drainage area of the basin in order to use the method. The dimensionless hydrograph has an area of 1,196.5 square units, a peak of 60 units, and a time base of 100 units. The computation method as given in the office procedure developed by Commons (1945) is indicated below. Equation 4 describes the means of computing the value of one square unit, V, of hydrograph area.

Table 1.--Snyder's coefficients developed by U. S. Army District, Fort Worth, Texas

River Basin	Stream and description of area	C_t	$640C_p$	$t^\dagger$
Neches	Angelina River Watershed above McGee Bend Dam	6.0**	320	6
	Neches River Watershed above Dam B	6.0**	310*	6
Trinity	Big Fossil Creek Watershed	.9	420	1
	Elm Fork Trinity--above Lewisville Dam	1.95	508	6
	Trinity River from confluence of Elm & West Forks to mouth of East Fork	1.8	500	6
	Trinity River from East Fork to head of Tennessee Colony Res.	2.9	460	6
	Cedar Creek above head of Tennessee Colony Res.	4.5	530	6
	Richland Creek Watershed from Bardwell and Navarro Mills Dam to head of Tennessee Colony Res.	3.0	530	6
Brazos	Brazos River above Whitney Dam	1.65	397	12
	Leon River--river mile 50.0 to Belton Dam	.7	500	3
	Lampasas River--above Stillhouse Hollow Dam	1.3	515	6
	San Gabriel River--North Fork above North Fork Damsite	.8	530	6
	Yegua Creek--above Somerville Dam	3.0	500	6
	Navasota River Watershed	3.0	475	6
Colorado	Hords Creek above Hords Creek Dam	1.0	600**	1
	Remainder of Pecan Bayou Watershed above Brownwood Dam	1.5	500	3
Guadalupe	Guadalupe River Watershed above Canyon Dam	.68	400	3
San Antonio	San Antonio River and tributaries in vicinity of San Antonio	.6*	500	1

* Minimum value

** Maximum value

† t is the duration of rainfall excess

$$V = \frac{\text{(Total Storm Runoff)}}{1{,}196.5} \tag{4}$$

The total storm runoff is usually converted to acre-feet. The computation of the value of 1 flow unit, f, is outlined in equation 5. The peak discharge is usually expressed in cfs.

$$f = \frac{\text{(Peak Discharge)}}{60} \tag{5}$$

The value of 1 time unit, t, is given in equation 6. C is a conversion factor

$$t = \frac{(V)\ (C)}{f} \tag{6}$$

for relating the units in such a way that t will be expressed in hours. Coordinates for the dimensionless hydrograph are given in the office manual along with a method for determining the peak discharge.

Another step in the utilization of the unit hydrograph method was taken by Brater (1940) when he applied the unit graph principle to small watersheds. The areas with which he worked varied in size from 4.24 to 1,876.7 acres. His investigation took place in the Southern Appalachians. The two main parts of the study were the selection of unit graphs and the preparation of distribution graphs as defined by Bernard (1935). Some applications of the distribution graphs were made in order to test the suitability of the theory to small watersheds.

An interesting item in Brater's paper was the definition given for the unit hydrograph. A unit hydrograph was defined as the hydrograph that would result from a unit storm. A unit storm was defined as "an isolated rainfall falling at an intensity greater than the infiltration capacity and having a duration equal to or less than the period of rise." This definition of unit hydrograph is also given by Brater (Wisler and Brater, 1959). Brater stated that a unit storm is defined by a rain whose duration is such that the period of surface runoff is not appreciably less for any rain of shorter duration.

As near as can be determined, Brater was the first to advance this definition of the unit hydrograph.

More recently, researchers such as Edson (1951), Nash (1959), and Dooge (1959) have developed equations for the shape of the unit hydrograph. The equations by Nash and Edson were compared by Gray (1961) to the equation for the incomplete or two-parameter gamma distribution. Gray made a study in order to develop data for the intermediate size drainage area of 1 to 50 square miles. Data for the study were obtained from 42 watersheds in Illinois, Iowa, Missouri, Nebraska, Ohio, and Wisconsin. Drainage areas ranged in size from 0.23 to 32.64 square miles. The definition of unit hydrograph used by Gray was the same as that advanced by Brater (Wisler and Brater, 1959). Gray stated in his paper that the duration of the unit storm varies from the period of rise for very small watersheds to only a fraction of the period of rise for large watersheds. A good summary of the criteria to be used in ascertaining the acceptability of data to unit hydrograph analyses is given in the paper. For each of the storms studied, distribution graphs as defined by Bernard (1935) were developed. An average distribution graph was then developed for each watershed studied, and a dimensionless graph was developed for each average distribution graph. Gray defined a dimensionless graph as one whose ordinates were expressed as ratios to the ordinate at 25 percent of the period of rise and whose abscissas were expressed as ratios to the period of rise. The dimensionless graph data were then fitted to a two-parameter gamma distribution. A statistical Chi-square test was used to determine the goodness of fit. Relations between the gamma distribution parameters and the selected basin characteristics were determined. Gray showed that for the data with which he worked the period of rise was a sufficient time parameter to be used in the analysis of the unit graphs.

A more recent investigation of unit hydrographs was made by Wu (1963) in Indiana. Wu referred to the work done by Edson (1951) and Nash (1959) concerning equations for the form of the unit hydrograph. Data were studied from 21 small watersheds that were distributed throughout the state of Indiana. A small watershed was defined by Wu to be any watershed less than 100 square miles in area. In any particular watershed Wu noted that the time to the peak did not vary significantly for different storms and therefore could be used as a hydrograph parameter. Data for isolated storms with high peaks and smooth recession curves were chosen for study. From the actual hydrograph data, dimensionless hydrographs were determined. A dimensionless hydrograph was considered by Wu to be one whose ordinates were evaluated by dividing the ordinates for the original hydrograph by the peak discharge and whose abscissas were determined by dividing the abscissas for the original hydrograph by the time to the peak. Typical dimensionless graphs for each watershed were developed. A study of the recession curves was made by plotting them on semilogarithmic paper and determining the best straight line or lines that fit the individual recession curve. From the straight line portion that defined the first part of the recession curve, Wu determined the storage coefficients used in his hydrograph analysis. A regression analysis was performed with the hydrograph parameters, time to the peak and storage coefficient, and the measurable basin characteristics. The important characteristics of the basin were found to be drainage area, length of the main stream, and representative slope of the main stream.

S-Curve Investigations

The S-curve is defined by Linsley (1958) as the "hydrograph which would result from an infinite series of runoff increments of 1 in. (inch) in t hours" where t is the duration of rainfall excess for the unit hydrograph. A detailed

discussion of the construction of the S-curve and its use in adjusting a unit hydrograph of one particular duration to the unit graph for another duration are also shown by Linsley.

In considering the history of the S-curve, it is felt that the S or summation curve hydrograph was first alluded to by Sherman (1932) when he showed that the hydrograph of runoff for a continuous uniform rain was S shaped. The so-called "S-curve method" was reported by Chow (1962) to have been suggested first by Morgan and Hullinghors (1939). This is the method by which unit hydrographs for various durations may be determined from a unit hydrograph of a given duration.

Linsley (1958) stated that "Commonly, the S-curve tends to fluctuate about the equilibrium flow." He also implied that this fluctuation may be greater if the rainfall excess duration is not the closest possible value to the correct one. As is shown in Figure 1, this fluctuation is caused by the fact that for other than the correct rainfall excess duration, the various rainfall histograms either overlap one another or there are gaps in the hyetograph. Linsley also stated that the "S-curve serves as an approximate check on the assumed duration of effective rainfall for the unit hydrograph." In his earlier book (Linsley, 1949), he stated "If the assumed duration is too short, there may be no indication in the shape of the S-curve, but the resulting derived unit hydrographs will of course be in error."

Most recently, the S-curve has been used by such workers as Henderson (1963) and Singh (1964) as a tool used in the derivation of the instantaneous unit hydrograph. The instantaneous unit hydrograph is the hydrograph of surface runoff that would result from 1 inch of rainfall excess falling in an infinitesimal time. It has been shown by both Henderson and Singh that the instantaneous unit hydrograph ordinates are proportional to the first derivative of the S-curve hydrograph. Singh (1962) tried polynomial and empirical curve

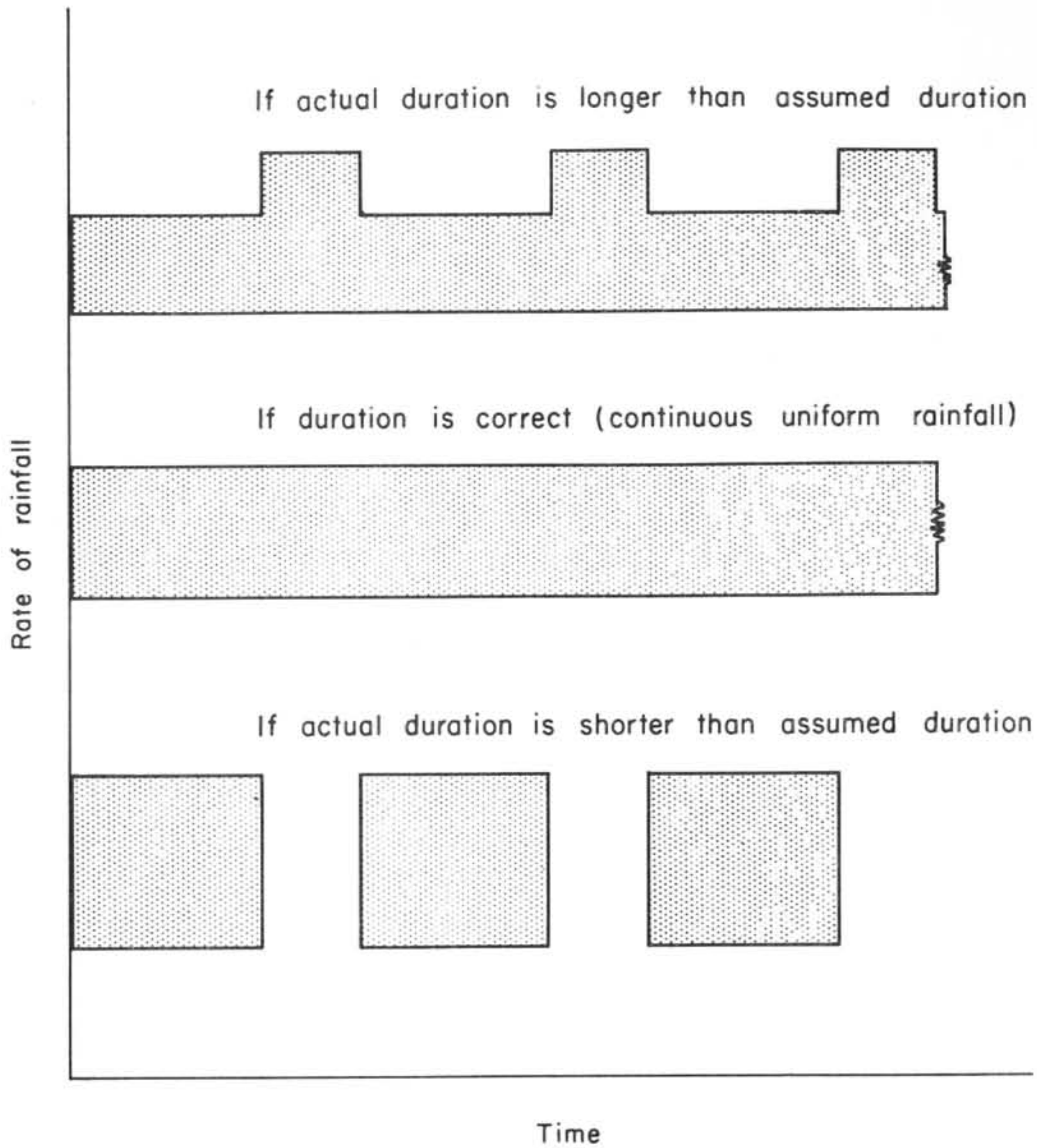

Figure 1
Influence of Incorrect Rainfall Excess Durations
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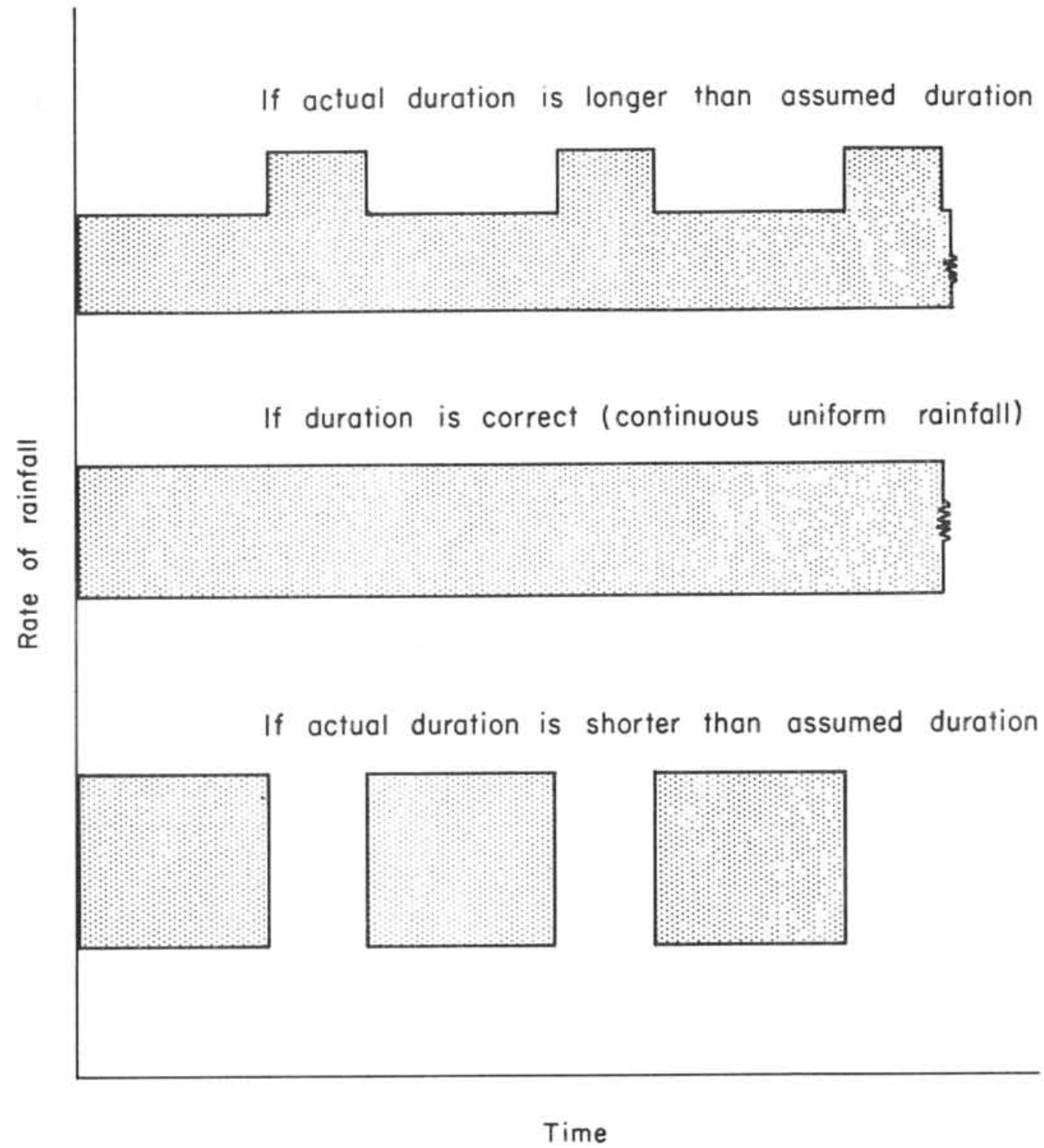

Figure I
Influence of Incorrect Rainfall Excess Durations
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fitting to actual S-curves in order to evaluate equations for the S-curve. These equations could then be used to determine the first derivative of the S-curve. The instantaneous unit hydrograph has certain advantages over the conventional unit hydrograph. One major advantage is that the lag time--from centroid of rainfall excess to peak of unit graph--equals the period of rise. Therefore, it is free of the variations in shape caused by duration and time distribution of rainfall excess.

PRESENT INVESTIGATION

Need for Study

Very little data are available regarding the characteristics of floods on small watersheds in Texas. There is also a decided scarcity of data concerning the analysis of flood hydrographs on small watersheds. This lack of information becomes more apparent when a person has to prepare design inflow hydrographs. Data that are needed for use in making spillway designs are usually not available. It is necessary, therefore, to evaluate design inflow hydrographs using empirical or synthetic methods. The relative merits of the several synthetic hydrograph procedures are not known because standards by which they might be checked usually do not exist. In many cases the results obtained when using the various synthetic procedures were considered to be unrealistic. However, little quantitative data were available with which a comparison might be made. Therefore, this study was initiated as a means of developing needed data.

After analysis of the rainfall and runoff data had begun, it became apparent that some relatively simple procedure was needed to estimate the rainfall excess duration. Methods that are currently available require either a considerable knowledge of the morphology, soil, and cover characteristics of an area or the use of some empirical means of estimating the rainfall excess

duration. It was decided in this study to investigate the feasibility of using the S-curve as a means for making this estimate.

Sources of Data

The U. S. Geological Survey in cooperation with the Texas Water Commission, the Soil Conservation Service, and other interested parties are making an intensive study of hydrologic data on 13 small watersheds in Texas. Study areas for the program were selected in an effort to gather data for which the results would be applicable within the region. According to the annual series of U. S. Geological Survey Open-File Reports concerning the small watershed program, there are three purposes of this study. They are:

1. To provide basic hydrologic data for small watersheds;

2. To provide hydrologic data through a series of extended climatic cycles that, when analyzed and interpreted, will lead to a better understanding of the effects of small floodwater-retarding structures on the water yield and mode of occurrence of surface water; and

3. To provide the Soil Conservation Service and other water-project planners with factual information for planning and designing land-treatment and flood-control measures.

Selection of Data

To avoid the problem of estimating the effect of upstream development, data from watersheds with no upstream development were analyzed. Data are available that have been gathered in the predevelopment state on only 4 of the 13 watersheds. The drainage areas on these four watersheds range from 17.6 to 75.5 square miles. One of these four areas was not included because topographic maps were not available. In addition one of the remaining three complete watersheds is only partially mapped. At the time this report is being written, only two of the watersheds remain in their undeveloped states. The three watersheds that were studied are located on a map of Texas in Figure 2. The gaging stations are designated Little Elm Creek near Aubrey, Mukewater duration. It was decided in this study to investigate the feasibility of using the S-curve as a means for making this estimate.

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Figure 2

Map of Texas Showing Location of Watersheds Studied

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Creek at Trickham, and Pin Oak Creek near Hubbard. Individual watershed maps indicating the density of instrumentation are shown in Figures 3, 4, and 5. Characteristics of the basins are shown in Table 2.

Table 2.--Watershed characteristics

Watershed name	Length of main stream (mi.)	Drainage area (sq. mi.)	Slope factor-main stream (ft./ft.)
Pin Oak Creek	7.92	17.6	0.00173
Mukewater Creek	19.2	70.0	.00228
Little Elm Creek	25.0	75.5	.00122

Description of Work Performed

For the most part, the data consisted of plotted hydrographs, mass rainfall and runoff curves, and tabulations of rainfall and streamflow data. In every case the streamflow data were gathered using recording gages. Some of the rainfall data were collected using recording gages, and some were collected using non-recording gages.

The first step in the analysis of the data was to subject the data to visual analysis. This involved looking over the plotted data in order to find the hydrographs that resulted from short-duration, high-intensity type storms. When these isolated storms had been found, the rainfall data from the recording gages were studied to determine the temporal and spacial distribution of the rainfall. After storms of relatively good distribution had been found, rainfall histograms were plotted. At this time an estimate of the rainfall excess duration was made. When the estimated rainfall excess duration had been compared with the period of rise of the hydrograph and found to be less than half of it, streamflow data were picked from the tabulations and hydrograph plots for card punching. These data were then used for the S-curve hydrograph analysis.

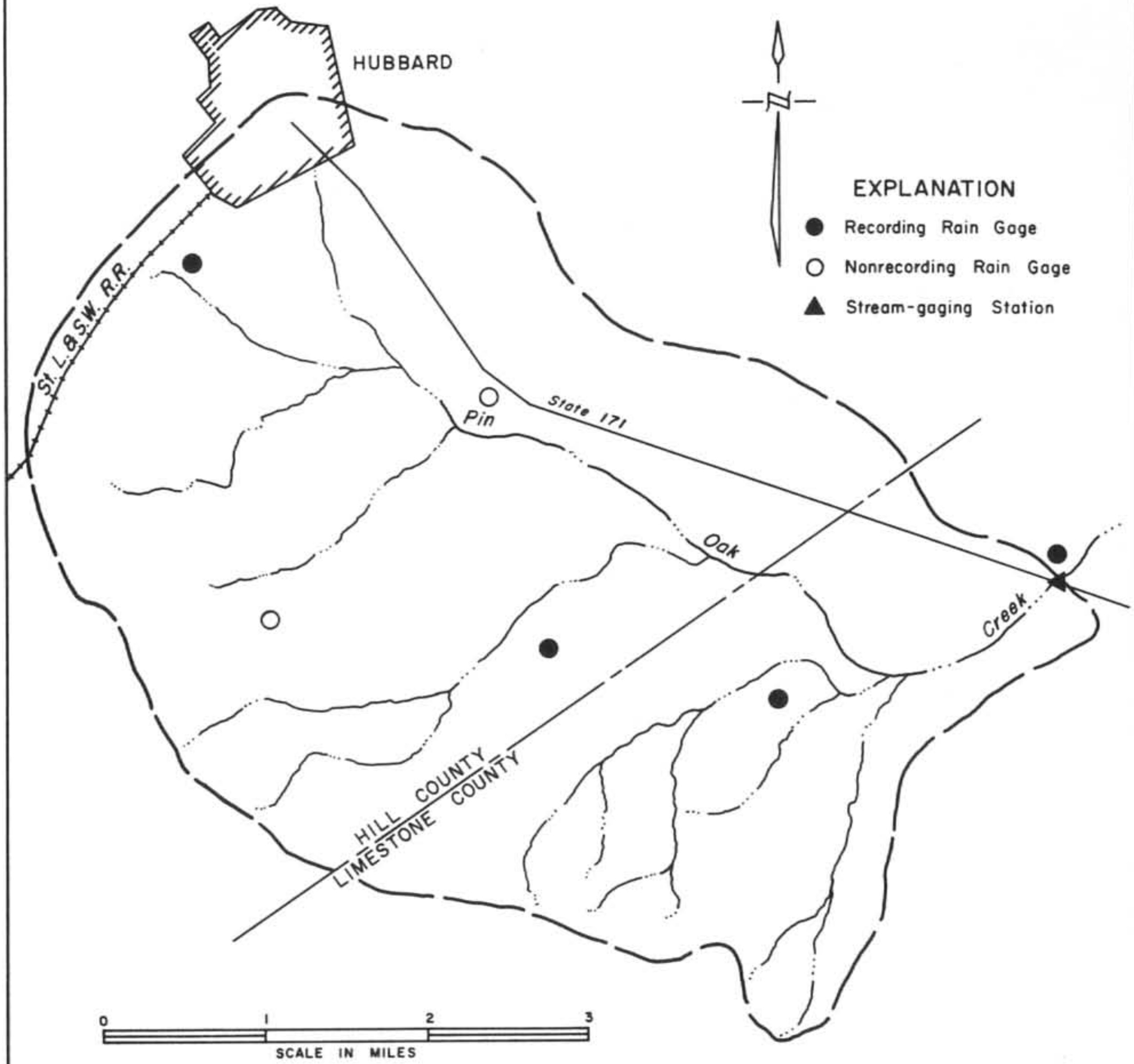

Figure 3

Map of the Pin Oak Creek Watershed Showing Density of Instrumentation

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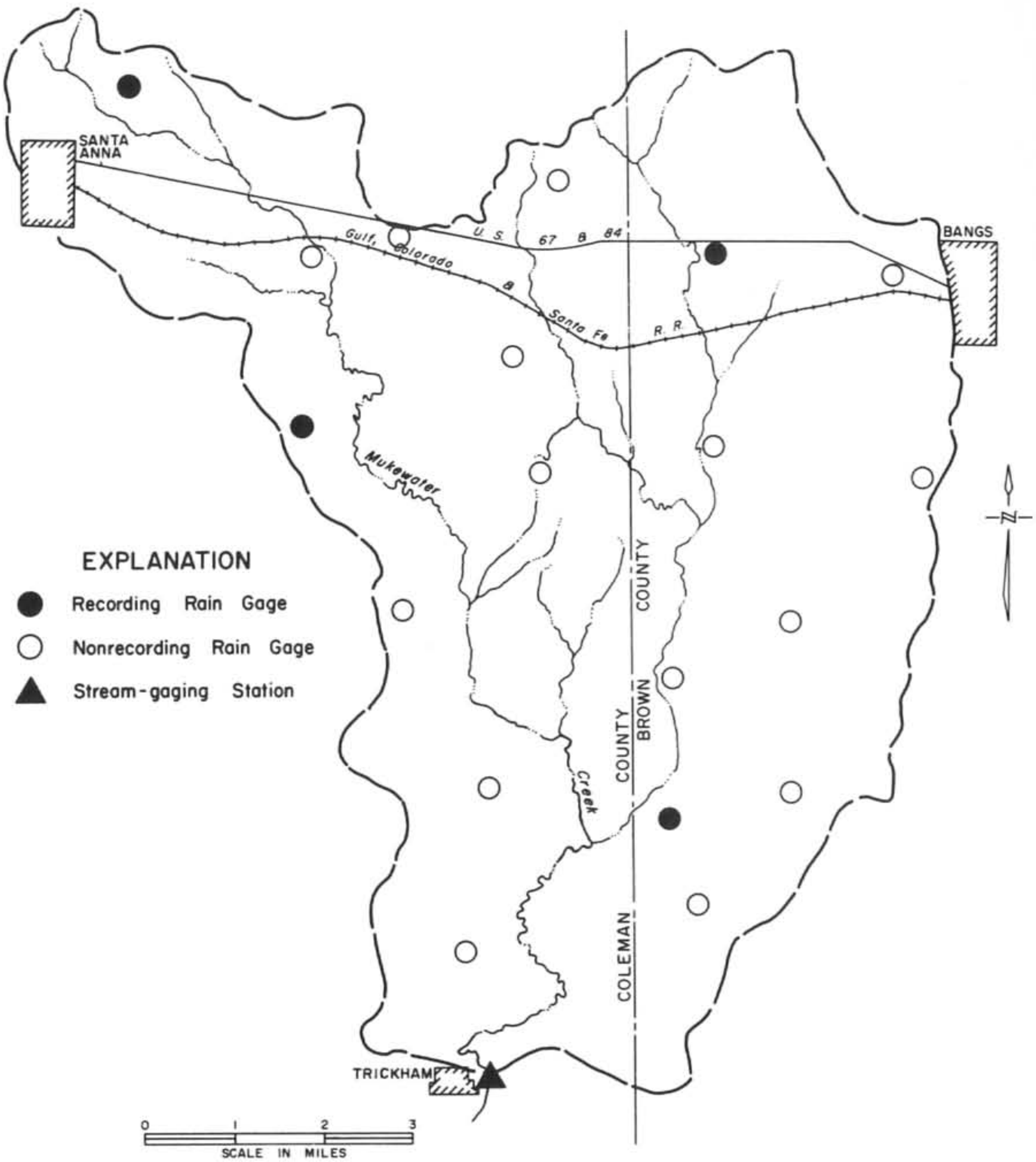

Figure 4

Map of the Mukewater Creek Watershed Showing Density of Instrumentation

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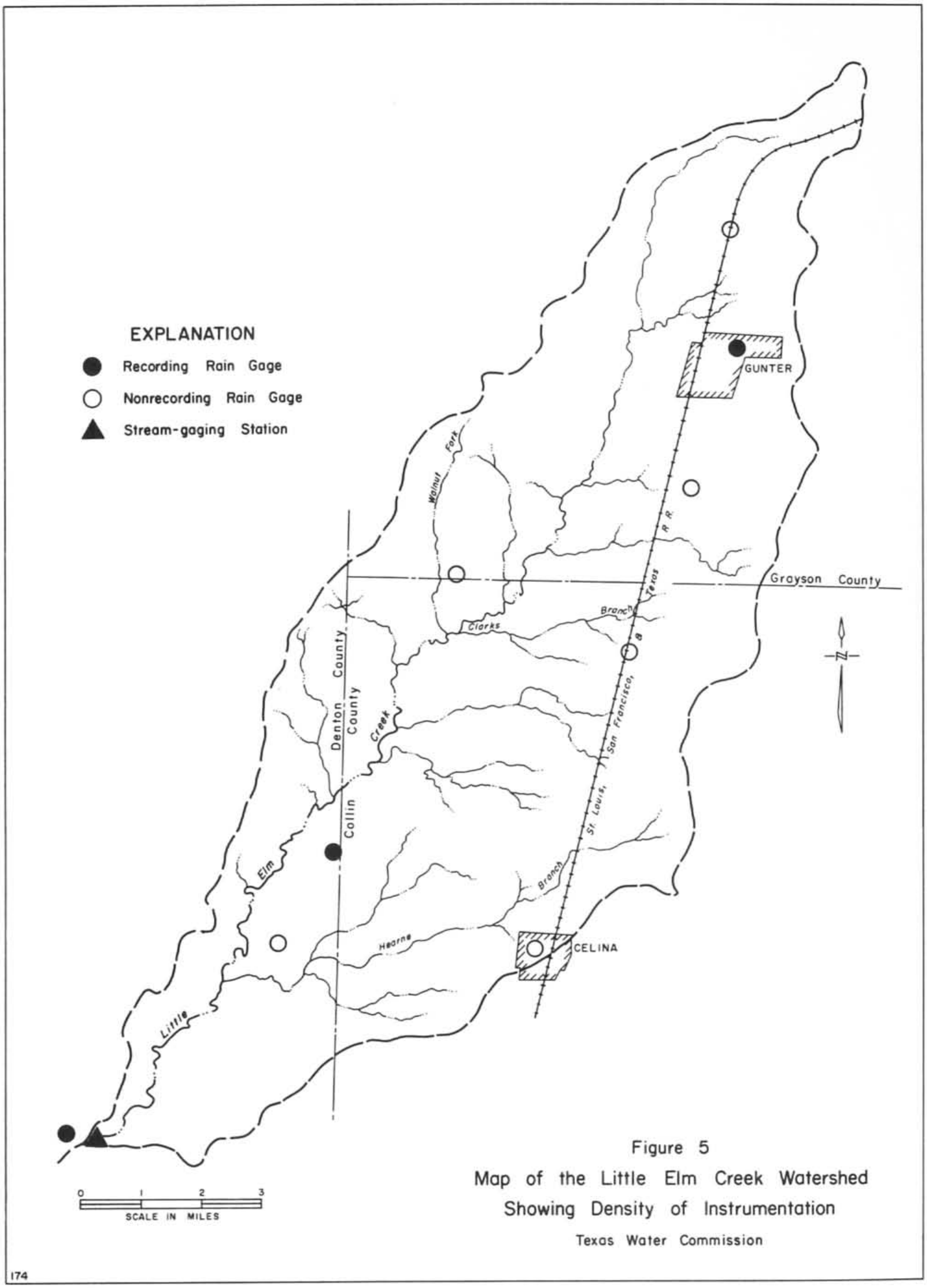

Figure 5
Map of the Little Elm Creek Watershed Showing Density of Instrumentation
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Although base flow was not a very significant item for the watersheds in this investigation every attempt was made to insure that only data for surface runoff were included in this study. The method chosen to estimate the amount of base flow was to plot the recession curves on semilogarithmic paper and to choose the point of deviation from a straight line as the end of surface runoff. A straight line from the beginning of rise to the surface flow end point was used to separate surface from base flow.

The S-curve analysis was used to make a better estimate of the rainfall excess duration. This method involved determining by trial and error the duration that resulted in the best S-curve. The best S-curve was defined as that curve which produced the best fit by a tenth-order polynomial and approached most nearly the theoretical equilibrium flow for that duration.

When the S-curve analysis had been completed and a rainfall excess duration had been selected, the program for unit graph analysis was employed. The first step was to attempt to eliminate time as a variable. This was done by adjusting all hydrographs by means of the "S-curve method" to a common duration--in this case 2 hours. In several instances this was shorter than the duration for the original unit graph. It was necessary to smooth the S-curve from which the adjusted hydrograph was to be computed. The smoothing was done to lessen the effect of the fluctuations that occurred in the computed S-curve. When these data were smoothed, the resulting S-curve had been reformed in such a way that unit graphs computed from it would be relatively smooth. The 2-hour unit graph was then computed and also smoothed. The data were then checked to insure that the area under the graphs represented 1 inch of direct runoff. Dimensionless hydrograph ordinates and abscissas were then computed using the peak discharge as the discharge base and the time to the peak as the time base. All the dimensionless graphs therefore peaked at the same point. The average dimensionless graphs for each watershed were then interpolated by eye.

Average 2-hour unit hydrographs were determined for each watershed from the average dimensionless hydrographs.

METHODS OF HYDROGRAPH ANALYSIS

Preliminary Considerations

As was stated previously, the purposes of this study were to develop a simple means of estimating the rainfall excess duration utilizing the S-curve and representative unit hydrographs and dimensionless graphs for each of the watersheds studied. The data with which the work was done were good in almost all cases. There were ample numbers of rain gages positioned in each watershed and at least two recording rain gages in each watershed. It was felt that with this coverage it would be possible to investigate methods of analysis and decide which were most feasible to be used in the investigation.

It was necessary first to choose which definition of the unit hydrograph to use. One viewpoint held by Linsley (1958) and others defines a unit graph in terms of the duration of runoff producing rainfall. The other view advanced primarily by Brater (1959) is that a unit graph is the result of a unit storm. A unit storm is defined as one whose duration is such that the period of surface runoff is not appreciably less for a storm of shorter duration. Brater states that such durations are shorter than the period of rise of the resulting hydrograph. From a study of the basic data it was determined that, although the durations of the storms that were selected for study in this investigation were usually much less than the periods of rise for the resulting hydrographs, there were still considerable variations in the shapes of the hydrographs. After a study of the rainfall records, it was concluded that the storms were as free from variation in distribution over the watershed as it was possible for them to be under natural conditions. It is believed that, as Brater has stated, there is a storm duration and rainfall distribution that defines a unit storm for a

particular basin. However, it is also felt that in order to use this definition much more data would be necessary in order to define the duration of the unit storm. The variation in actual hydrograph shapes was believed to be caused by variations in the actual storms. Since this variation occurs continually in nature, it was felt that it would be better to develop unit hydrographs using the definition of Linsley (1958) and others in which consideration is made for the varying duration of rainfall excess. It was also decided to compute coordinates of dimensionless hydrographs whose ordinates would be expressed as the unit graph discharge divided by the peak discharge and whose abscissas would be expressed as the unit graph abscissas divided by the period of rise of the unit hydrograph. The dimensionless graphs would be used to aid in the hydrograph averaging and to compare with synthetic graphs expressed in dimensionless form. The variations in storm type were averaged by developing an average unit graph.

To work through the procedure outlined above, it was decided to utilize two separate computer programs. The first program was concerned with the S-curve analysis. The results of this program were used along with the rainfall data to estimate the duration of runoff producing rainfall. When the best possible duration had been picked, the coordinates of the 2-hour unit and dimensionless graphs were computed.

S-Curve Methods

The first program utilized the S-curve analysis as a tool for making the best possible estimate of the rainfall excess duration. The method used to compute the unit graph, on which the computed S-curve was based, is outlined in the section on unit graph analyses, which follows. The method utilized to compute the S-curve was a simplified form of the one given by Linsley (1958). It is the one presented by the U. S. Corps of Engineers (1959) in their recent

hydrologic design manual. It has been pointed out that, if an incorrect rainfall excess duration is assumed, an S-curve will be computed, which will fluctuate about an equilibrium flow rather than approach a constant value. The value of rainfall excess duration that caused the least fluctuation of the S-curve was determined by trial. It was decided to investigate fitting polynomial equations to actual S-curves using the method of least squares.

Polynomial fitting was not found to be completely satisfactory for natural S-curve data. The problem that developed in using polynomial fitting was that the curves that characterized the polynomial fits exhibited a cyclic variation and did not describe the non-decreasing function, which is characteristic of a theoretical S-curve shape. Singh (1962, 1964) reported that a sigmoid curve as described by Davis (1962) was found to fit the S-curve acceptably. The sigmoid curve analysis was tried in one instance in this investigation. It seemed that in this particular instance the sigmoid curve did not fit the actual S-curve satisfactorily. The sigmoid curve seemed to be better suited to fitting symmetrical S-curves. Naturally occurring S-curves are unsymmetrical since they occur from unit graphs, which are skewed.

Because of the amount of data and the difficulty of performing the proposed computations, the decision was made to use a digital computer. A consideration favoring the use of the computer and polynomial fitting was the availability of a computer program for least squares fitting of polynomials. The computer that was used was the Control Data Corporation 1604 at The University of Texas Computation Center. The library program (Raney, 1962) for least squares curve fitting uses a computational algorithm (Forsythe, 1957) built around recurrence relations, and does not resort to the more traditional matrix techniques. The library routine was used as a subroutine in the main program for computing S-curves. The program, for which a flow diagram is shown in Figure 6, was used to compute the unit hydrograph and three trial S-curves for

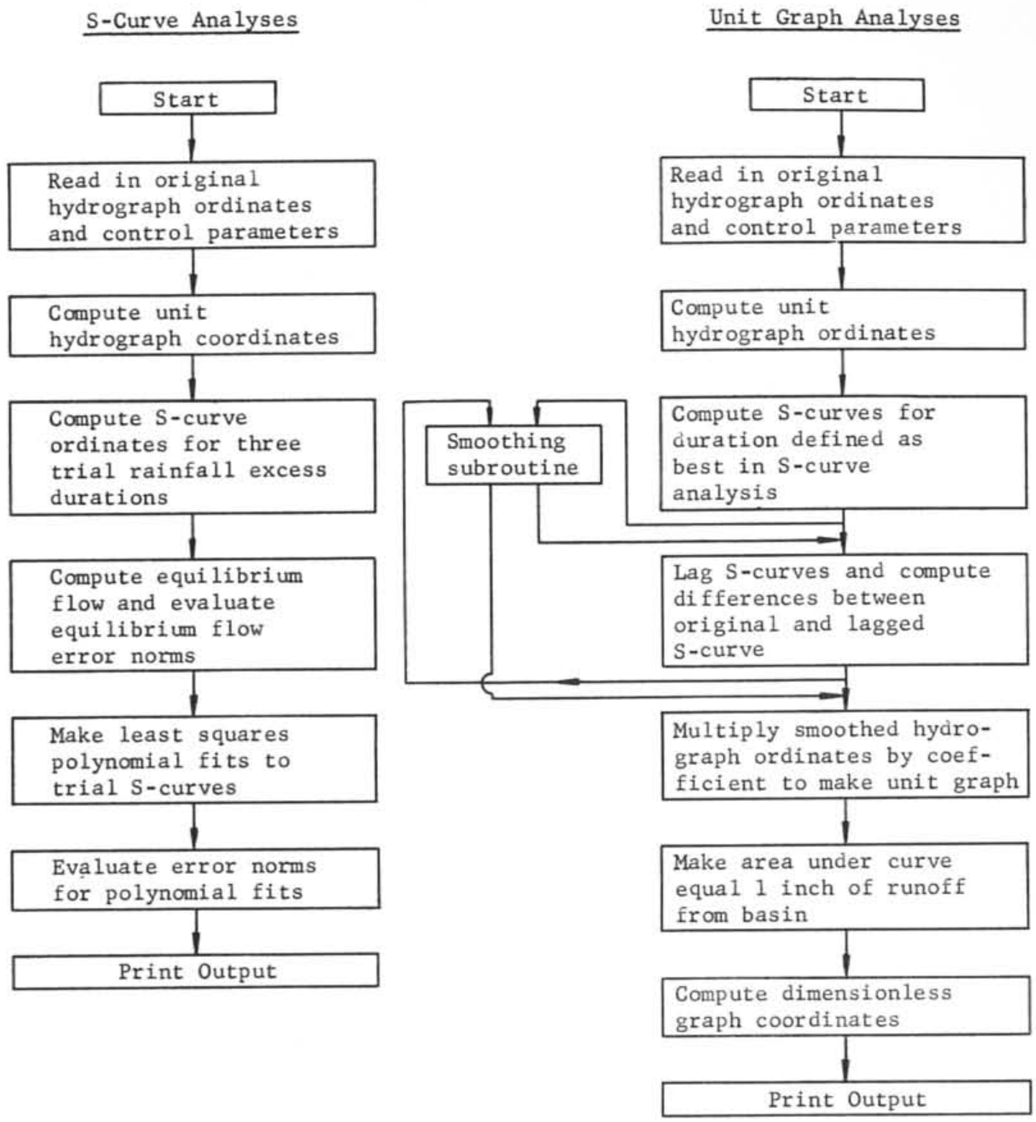

Figure 6

Simplified Computer Flow Diagram for S-Curve and Unit Graph Analyses

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a given hydrograph. The three S-curves were those for three successive trial durations of rainfall excess each separated by the time interval at which runoff ordinates were picked for data punching. The beginning trial duration was estimated from actual rainfall records. Successive runs were made until an acceptable value was obtained.

When seeking a method for fitting a polynomial equation to the computed S-curve points, a variation in the method described in the preceding paragraph was used. The technique involved the fitting of a polynomial to a curve whose ordinates were defined as the computed S-curve ordinates through the period of surface runoff and as the theoretical equilibrium flow for abscissas greater than the period of surface runoff. This particular method was not used because a significant improvement in the fit was not effected.

The actual evaluation of the acceptability of an S-curve for a particular duration of rainfall excess was accomplished using error norms, which were evaluated in the program, and the actual rainfall data. Two different types of error indications were used.

The first set of error norms indicated how well the equilibrium portion of the S-curve compared with the theoretical equilibrium flow. Three measures of the fluctuation about the equilibrium flow were evaluated. Equations representing the methods for computing these norms are shown below. Equation 7 indicates the method used to compute the theoretical equilibrium flow, Qe.

$$Qe = \frac{645.6A}{t} \tag{7}$$

The drainage area in square miles is represented by A. This equation, given by Linsley (1958), represents a uniform runoff rate of 1 inch every t hours where t is the duration of rainfall excess.

$$EN_1 = \sqrt{\frac{\sum_{i=1}^{N_1} (SC_i - Qe)^2}{N_1}} \tag{8}$$

Error norm one, EN_1, is equal to the square root of the summation of the squares of the differences between the computed S-curve points, SC_i, and the theoretical equilibrium flow value, Qe, divided by the number of abscissa points, N_1, greater than the period of surface runoff. The second error norm was computed according to equation 9. This value,

$$EN_2 = \frac{\sum_{i=1}^{N_1} \sqrt{(SC_i - Qe)^2}}{N_1} \tag{9}$$

EN_2, represents a measure of the average difference irrespective of whether the difference is positive or negative. The final norm, EN_3, used to evaluate the comparison of the computed points with the theoretical equilibrium flow is shown in equation 10. This value

$$EN_3 = \frac{\sum_{i=1}^{N} (SC_i - Qe)}{N_1} \tag{10}$$

when considered with error norm two is indicative of whether the computed S-curve points lie above, below, or fluctuate about the theoretical equilibrium flow.

The four measures used to evaluate the amount of fluctuation of the computed S-curve points are shown in equations 11 through 14. Equation 11 represents the sum of the differences between the fitted S-curve points, SCF_i, and the computed S-curve points, SC_i.

$$EN_4 = \sum_{i=1}^{N} (SCF_i - SC_i) \tag{11}$$

The fifth value, EN_5, indicates the average deviation irrespective of whether they are positive or negative. The value,

$$EN_5 = \frac{\sum_{i=1}^{N} (SCF_i = SC_i)}{N} \tag{12}$$

N, equaled the total number of computed S-curve points. Equation 13 represents the computation of the sixth error norm, EN_6, which is akin to EN_1.

$$EN_6 = \sqrt{\frac{\sum_{i=1}^{N} (SCF_i - SC_i)^2}{N}} \tag{13}$$

The seventh error norm shown in equation 14 equaled the maximum absolute value of any single difference between the fitted and computed S-curve points.

$$EN_7 = \max \; (SCF_i - SC_i) \tag{14}$$

The program, although used as an aid to judgment in selecting the best duration, was not the sole basis for picking the duration. Although polynomial fitting was not found entirely satisfactory for fitting curves to S-curves, it was found to be sufficiently accurate to be used as a method for picking the correct rainfall excess duration.

The length of time interval at which hydrograph ordinates are chosen is felt to be important in the determination of the best rainfall excess duration by the method under investigation. No attempt was made to make quantitative estimates of what the optimum time interval should be. However, on the basis of these observations, it can be concluded that the time interval should be probably never greater than one-third the final duration selected.

One of the interesting items brought out in this study was that if the rainfall excess duration exactly equalled the time interval at which hydrograph ordinates were picked, the resulting S-curve would not fluctuate. This therefore would give the indication at first glance that the S-curve that was computed was the S-curve for the correct rainfall excess duration. If the time interval is chosen small, this is rarely the case.

Unit Graph Methods

When the best rainfall excess duration had been determined, the unit graph analysis was employed. The unit graph analysis involved computing the ordinates of the unit graph for the duration of runoff producing rainfall and using these unit graph ordinates to compute S-curve ordinates. The S-curve was then used as a means for converting the various unit graphs for separate durations to unit graphs for the particular duration of 2 hours chosen for this study. The method for adjusting unit graphs of one duration to those for another duration was that method discussed by Linsley (1958). The 2-hour unit graphs were converted to dimensionless graphs as indicated previously in this report. The flow diagram for the computer program describing this method is shown in Figure 6.

The method proposed by Linsley (1958) was used to compute ordinates for the unit hydrographs. This method involves the computation of the actual runoff by numerically integrating the area under the gaged hydrograph and dividing each of its ordinates by the amount of runoff in inches. Since the volume of runoff is a critical factor, methods of numerical integration were studied. Simpson's Rule, a second order approximation, and the cubic rule, a third order approximation, were studied. The method proposed by Linsley is in effect a first order approximation or the trapezoidal method of numerical integration with the beginning and ending ordinates equal to zero. For various hydrograph shapes, it was found that the trapezoidal rule was sufficiently accurate to be used throughout this study.

One of the problems encountered in the program for unit graph analysis was obtaining an S-curve for the duration shifting operation that would be free enough from fluctuations to assure a resulting smooth unit graph. As was stated in the previous section, the fitted polynomials were not satisfactory to be used in the shifting operation. It was discovered that if the theoretical

equilibrium flow was introduced as a constant value on the actual S-curve after the period of surface runoff was past, the resulting S-curve was considerably smoother than the curves described by the fitted polynomials. Only the upper portion of the curves exhibited fluctuation. It was decided to use a smoothing method on the curve. The method used was the one presented by Scarborough (1958) for smoothing data plots that exhibited sharp changes in curvature. A parabolic curve is passed through each successive five points using this method. This particular method was chosen because it could be used on both the S-curve and the final unit graph. The program was set up so that an iterative process would be followed until the difference between the original and final values for a particular smoothing run were not greater than a particular tolerance level set in this study as 10 cfs. The program was also designed in such a way that a maximum of only 10 iterations would be performed in a particular smoothing operation. This same procedure was used in smoothing unit graph ordinates. It was seldom necessary for the program to iterate more than three times. The resulting shifted unit and dimensionless graphs were still not completely smooth, but they were greatly improved. It was also felt that little bias was imposed on the data by this method.

PRESENTATION OF RESULTS

S-Curve Analyses

The upper portions of the S-curve for four selected storms are shown in Figures 7, 8, 9, and 10. S-curves for all three watersheds studied are included. In the center of each figure, the S-curve is shown for the duration found to be best in the S-curve analysis. S-curves for durations that were half an hour shorter and half an hour longer than the best duration are shown on each figure also. On each individual S-curve plot, the actual computed S-curve points are indicated. A smooth reference curve is plotted as a yellow line. This reference

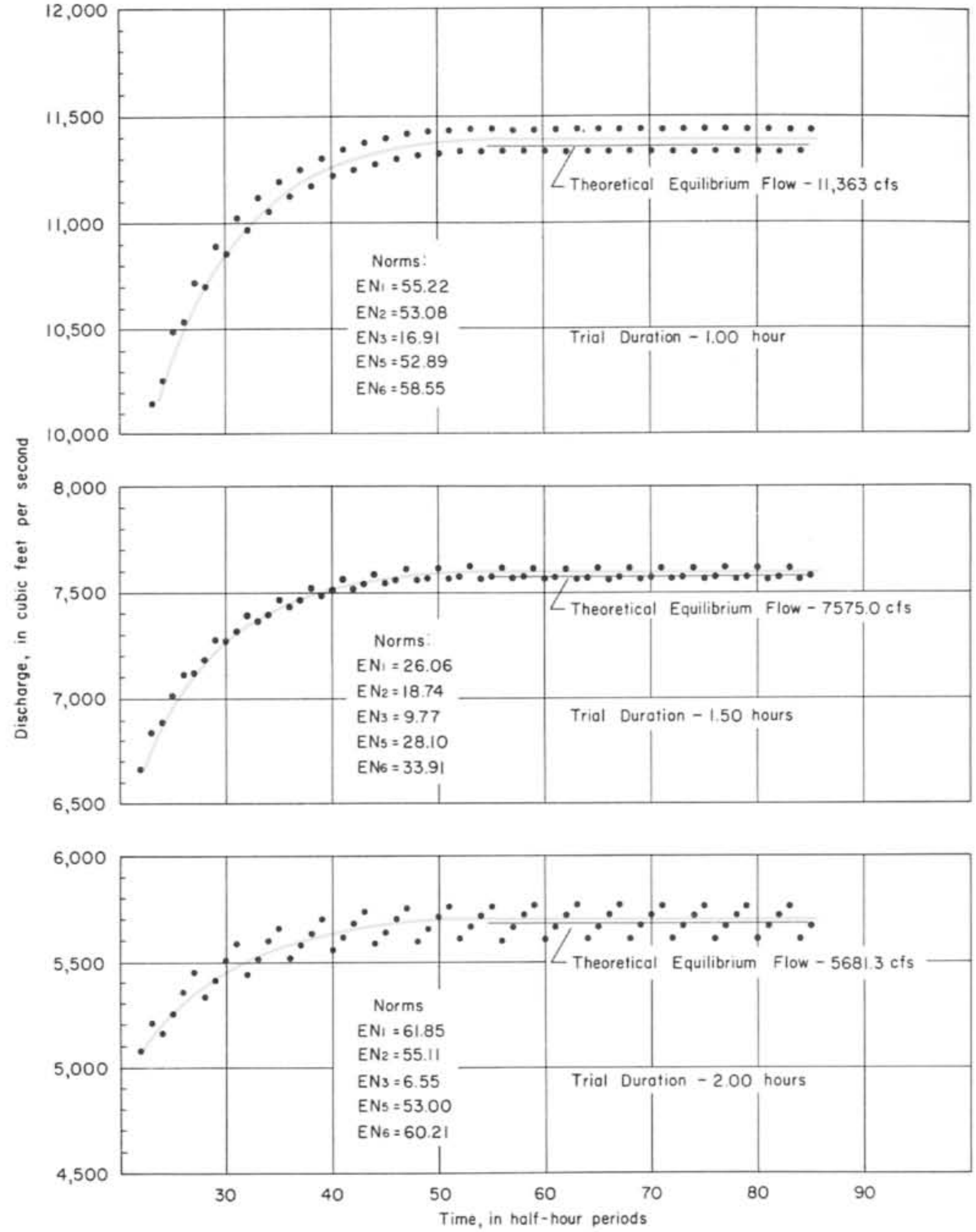

Figure 7

Upper Portion of S-Curves, Pin Oak Creek Watershed, Flood of February 1, 1957

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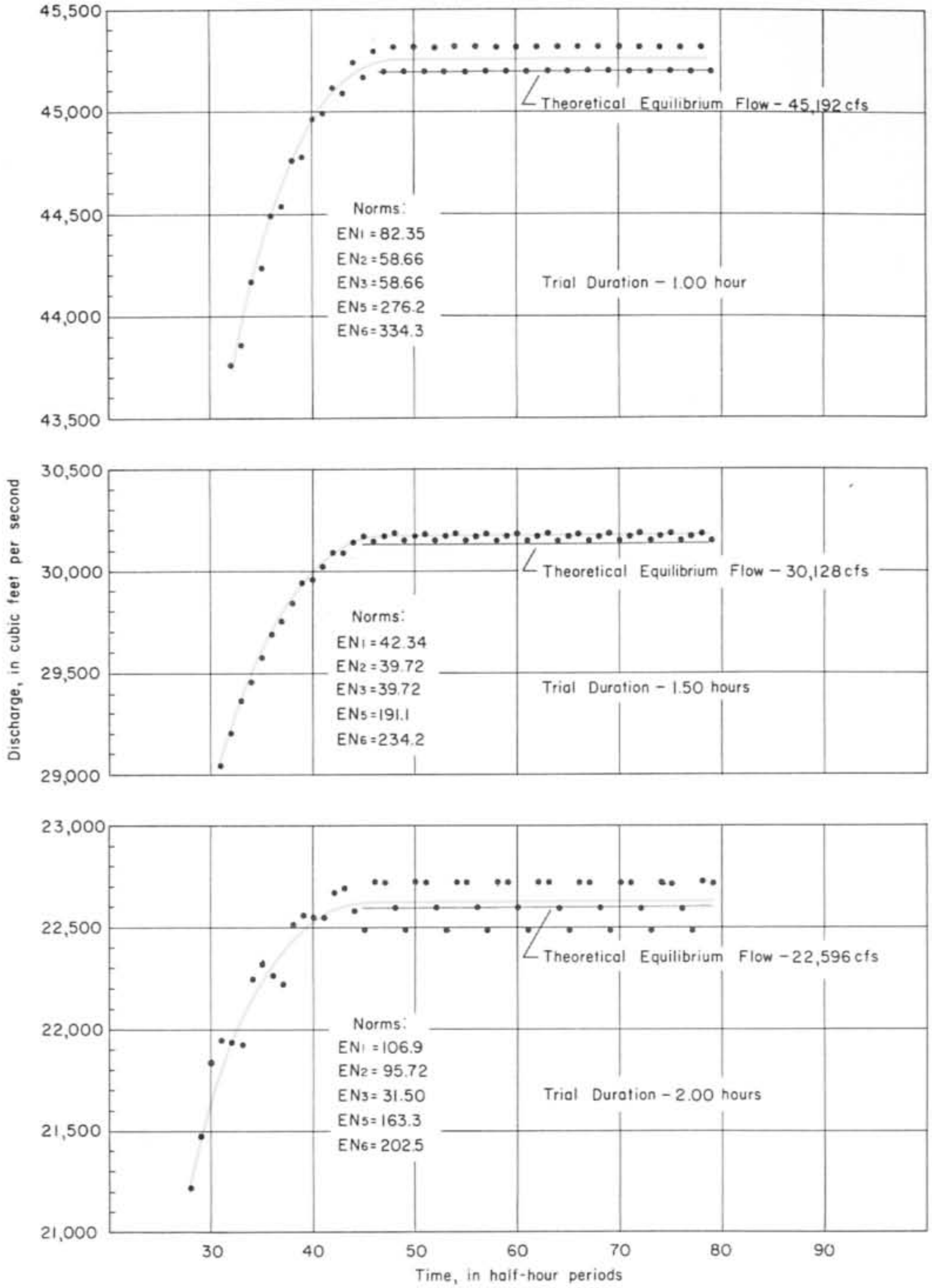

Figure 8

Upper Portion of S-Curves, Mukewater Creek Watershed, Flood of May 27, 1957

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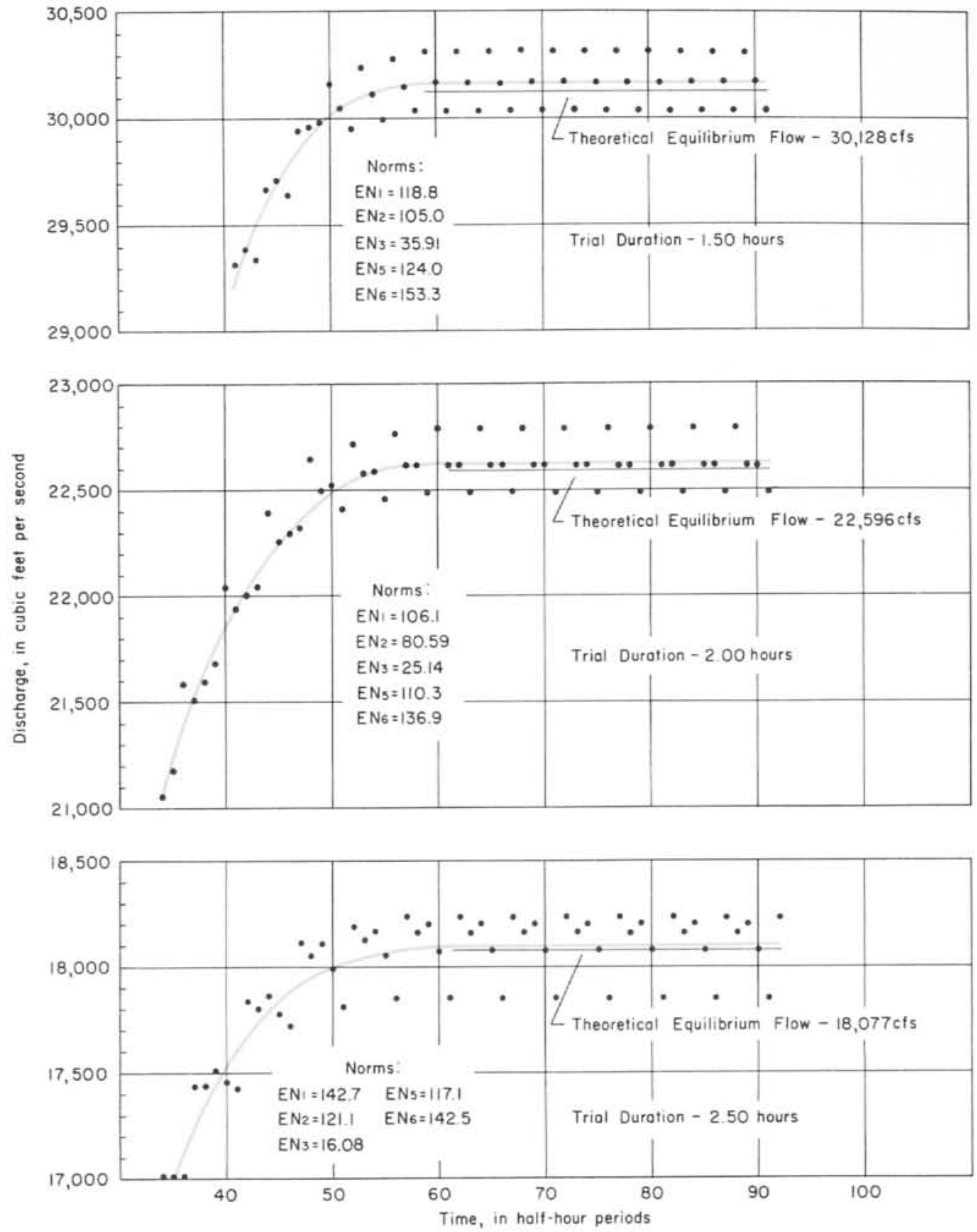

Figure 9

Upper Portion of S-Curves, Mukewater Creek Watershed, Flood of February 16, 1961

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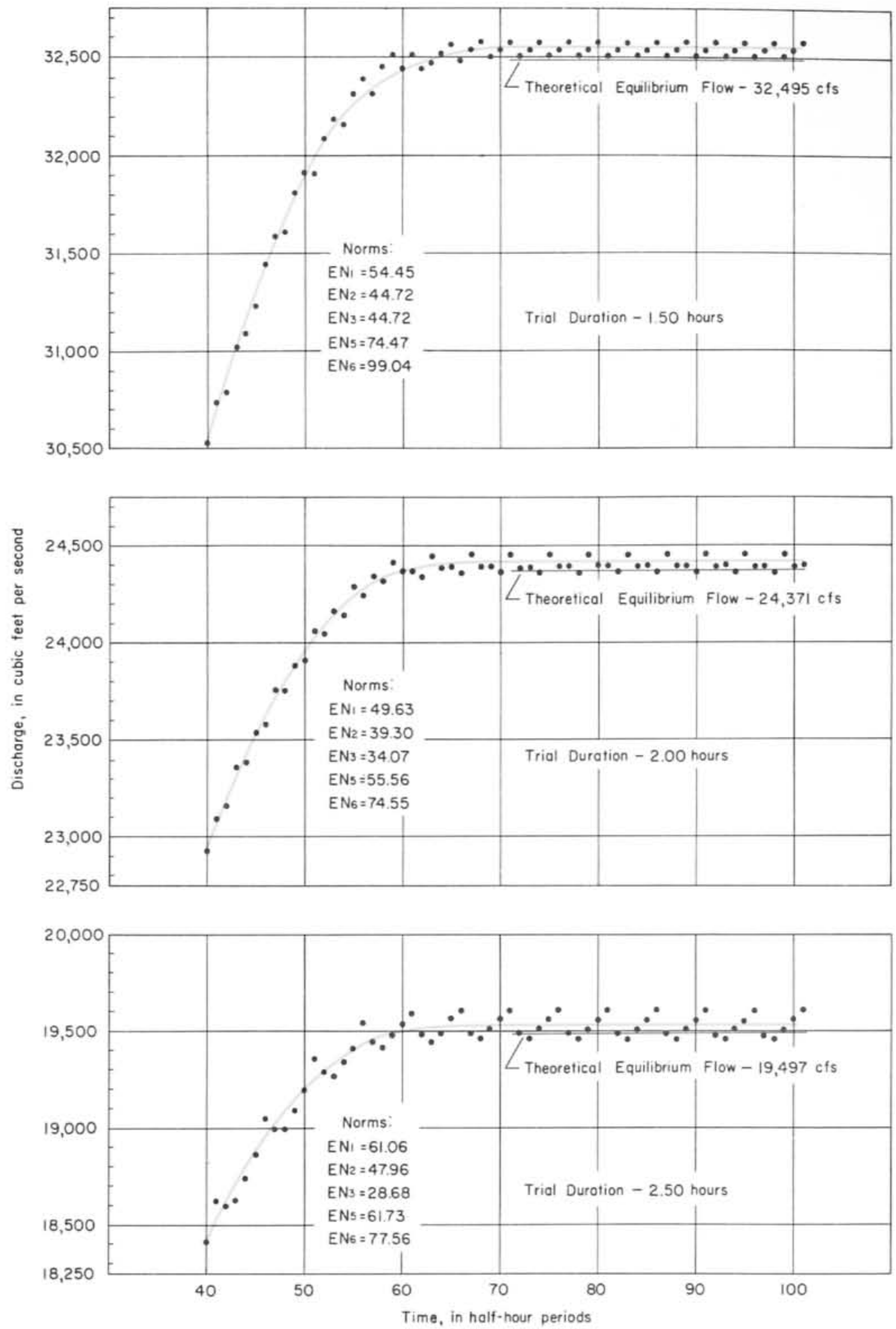

Figure 10

Upper Portion of S-Curves, Little Elm Creek Watershed, Flood of July 19, 1960

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curve should not be confused with the smooth S-curve computed in the unit graph analysis. The theoretical equilibrium flow is shown as a red horizontal line on each S-curve plot. Selected norms that were used to determine which duration defined the best S-curve are shown below each S-curve. Each norm is designated by the same subscript that was used on pages 23, 24, and 25 of this report.

The norms designated with subscripts 1, 2, and 3 represent the comparison of computed S-curve points with the theoretical equilibrium flow. The norm designated with subscript 1 is the root mean square of the differences between the computed S-curve points and the theoretical equilibrium flow. It was the most important value used in the comparison of the computed points with the theoretical equilibrium flow. The norms subscripted 2 and 3 were useful in determining whether the theoretical equilibrium flow lay above or below the computed S-curve points. If all computed points fell above the equilibrium flow, these two norms were equal and positive. If the points lay below the theoretical equilibrium flow, both values were equal but of different sign. The computed points were scattered about the equilibrium flow when the norms were unequal.

Those norms subscripted 5 and 6 are indicative of the polynomial fits. They could be used only as qualitative measures of the scatter of computed points throughout the S-curve. Generally, curves were selected as best when these norms indicated lower values for one duration as opposed to others. Because of the cyclic variation of both the computed points and the polynomial fits, these norms were not lower in every case for the best duration.

Unit Graph Analyses

Pertinent data on all analyzed hydrographs are shown in Table 3. Selected 2-hour unit hydrographs for each of the study areas are shown in Figures 11, 12, and 13. With the exception of one graph for Pin Oak Creek and two

Table 3.--Hydrograph characteristics

Drainage basin	Date	Original hydrograph					Unit hydrograph	Derived two-hour unit hydrograph		
		Rainfall excess duration (hrs.)	Runoff (in.)	Period of rise (hrs.)	Peak discharge (cfs)	Time base (hrs.)	Peak discharge (cfs)	Period of rise (hrs.)	Peak discharge (cfs)	Time base (hrs.)
Pin Oak Creek	Feb. 1, 1957	1.5	0.363	5.0	487.5	27.0	1,342	5.5	1,298	27.0
	May 11, 1957	1.0	1.06	6.0	2,227	24.5	2,091	6.5	1,985	25.0
	Aug. 24, 1958	1.5	2.38	6.0	4,299	17.0	1,809	6.0	1,810	17.5
	May 11, 1959	1.5	1.14	6.5	1,691	28.5	1,481	6.5	1,459	28.5
	June 24, 1959	1.0	2.43	5.0	4,047	21.5	1,665	6.0	1,635	22.0
Mukewater Creek	June 15, 1955	.5	.376	7.0	2,070	20.0	5,501	8.0	5,360	20.5
	May 11, 1957	1.0	.679	11.0	2,547	23.0	3,753	11.0	3,743	23.5
	May 27, 1957	1.5	.320	11.5	1,470	24.0	4,594	11.5	4,504	24.0
	June 22, 1958	1.5	.054	3.0	387	19.0	7,110	4.0	6,609	19.0
	June 4, 1959	1.5	1.17	7.0	5,700	23.5	4,855	7.5	4,813	23.0
	Feb. 16, 1961	2.0	.094	6.0	492.0	30.0	5,212	6.0	5,250	29.5
Little Elm Creek	July 17, 1959	2.0	.146	15.0	444.0	40.0	3,032	15.0	3,024	39.5
	Oct. 4, 1959	3.0	.623	11.0	1,641	70.0	2,632	11.0	2,688	51.0
	Nov. 4, 1959	3.0	.679	13.5	1,928	74.5	2,841	13.0	2,882	58.5
	Dec. 16, 1959	7.0	.523	18.5	1,292	61.0	2,472	15.5	2,841	46.5
	July 19, 1960	2.0	.113	6.5	547.1	35.0	4,863	6.5	4,850	35.5
	Apr. 28, 1962	6.0	.521	20.0	1,200	59.5	2,305	16.0	2,496	37.0

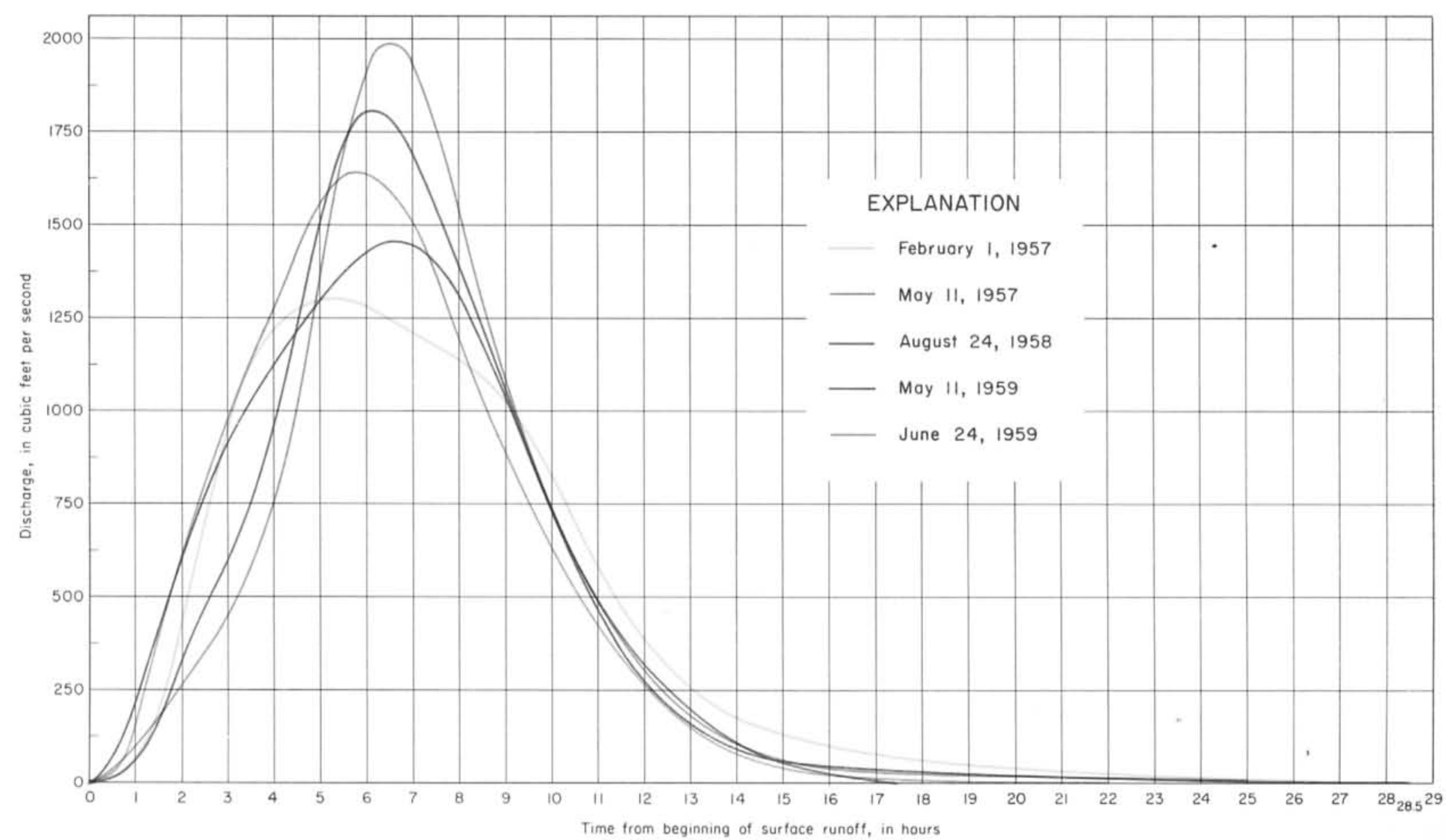

Figure 11
Two-Hour Unit Hydrographs, Pin Oak Creek Watershed
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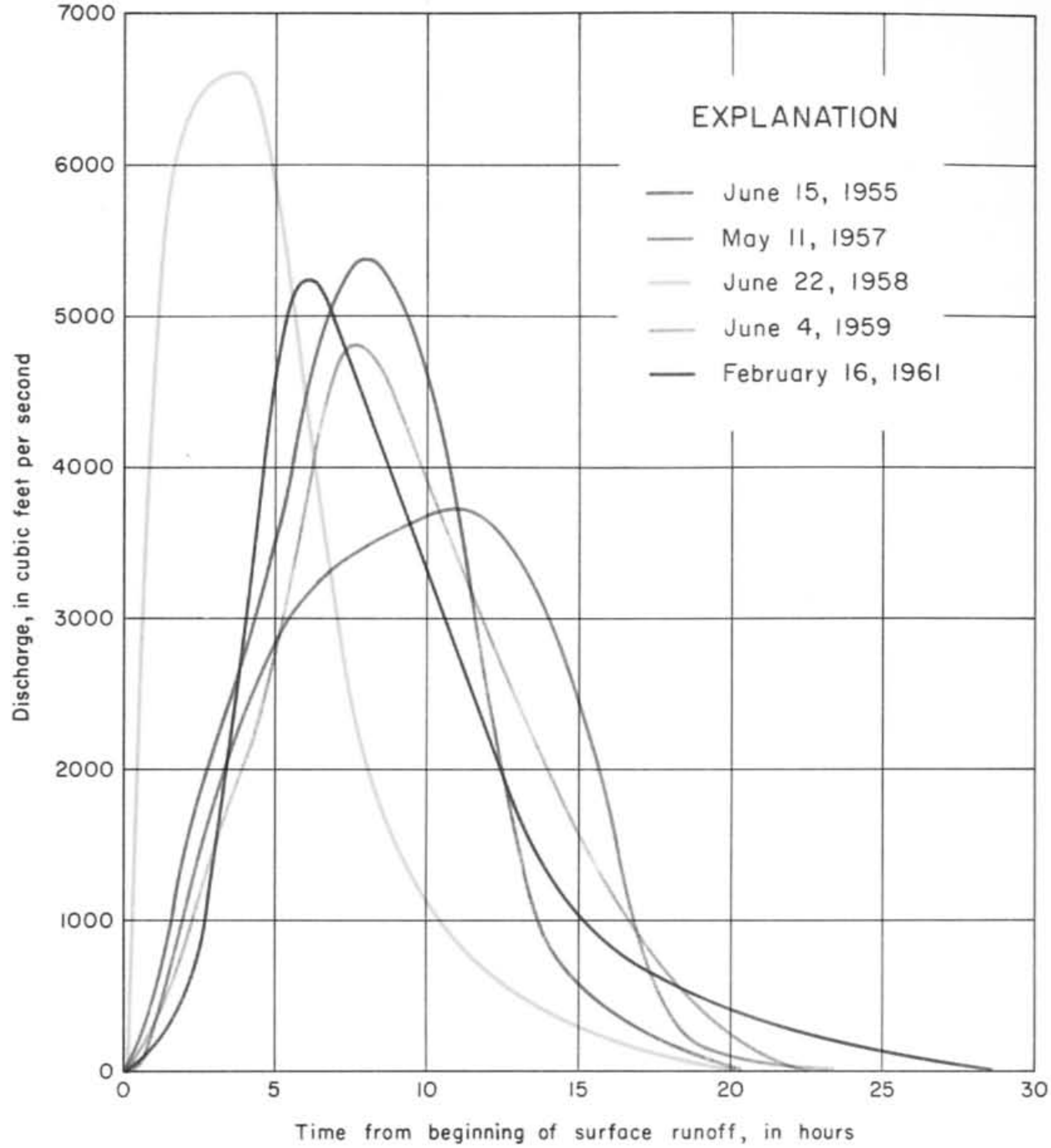

Figure 12

Two-Hour Unit Hydrographs, Mukewater Creek Watershed

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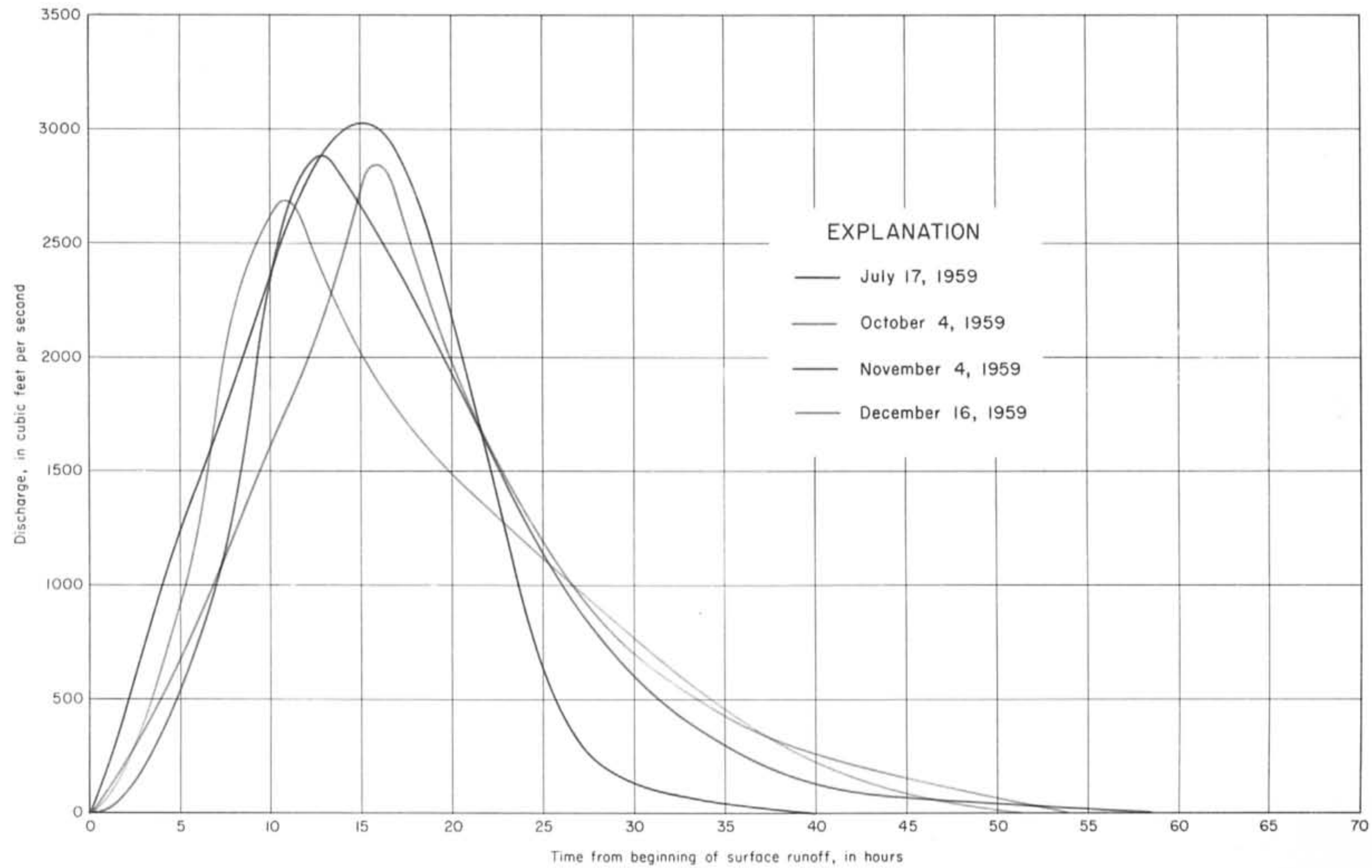

Figure 13

Two-Hour Unit Hydrographs, Little Elm Creek Watershed

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graphs for Mukewater Creek, the plotted unit graphs for each basin compared well. Causes for the variations in these curves will be considered in the discussion of the results. Only minor smoothing was necessary in the plotting of the final 2-hour graphs.

Two-hour unit graphs for three of the analyzed original hydrographs were not plotted. Two of the hydrographs for Little Elm Creek were not included in Figure 13. The unit graph for the storm of July 19, 1960 was not included because the rainfall was not well distributed over the basin. The storm of December 16, 1959 resulted in a hydrograph that when adjusted to a 2-hour unit hydrograph exhibited erratic fluctuations and therefore was not plotted. The unit graph for the storm of May 27, 1957 on Mukewater Creek was not included because its shape was considered to be unrepresentative of the area. It exhibited a delayed peak, a gradual rising limb, and a sharp falling limb.

Dimensionless hydrographs were computed for each of the plotted 2-hour unit graphs, and are shown for each watershed in Figures 14, 15, and 16. The ordinates and abscissas of the dimensionless graphs represent the ratios of unit graph ordinates and abscissas to the unit graph peak discharge and period of rise respectively. The factor that seemed to cause the major variation in the shape of the dimensionless hydrograph was the period of rise. The influence of the period of rise is indicated by the dimensionless graph for Mukewater Creek of June 22, 1958. A short period of rise caused the dimensionless graph to deviate from the other dimensionless plots. There is a pronounced deviation in the rising limb of the dimensionless graph. The falling limb is long even though time base for the 2-hour unit graph was the shortest.

Average dimensionless 2-hour unit hydrographs were interpolated by eye for each watershed studied. Plots of these curves are shown in Figure 17. The coordinates from which these curves were plotted are shown in Table 4. The average dimensionless graphs were developed both as a means of comparison with

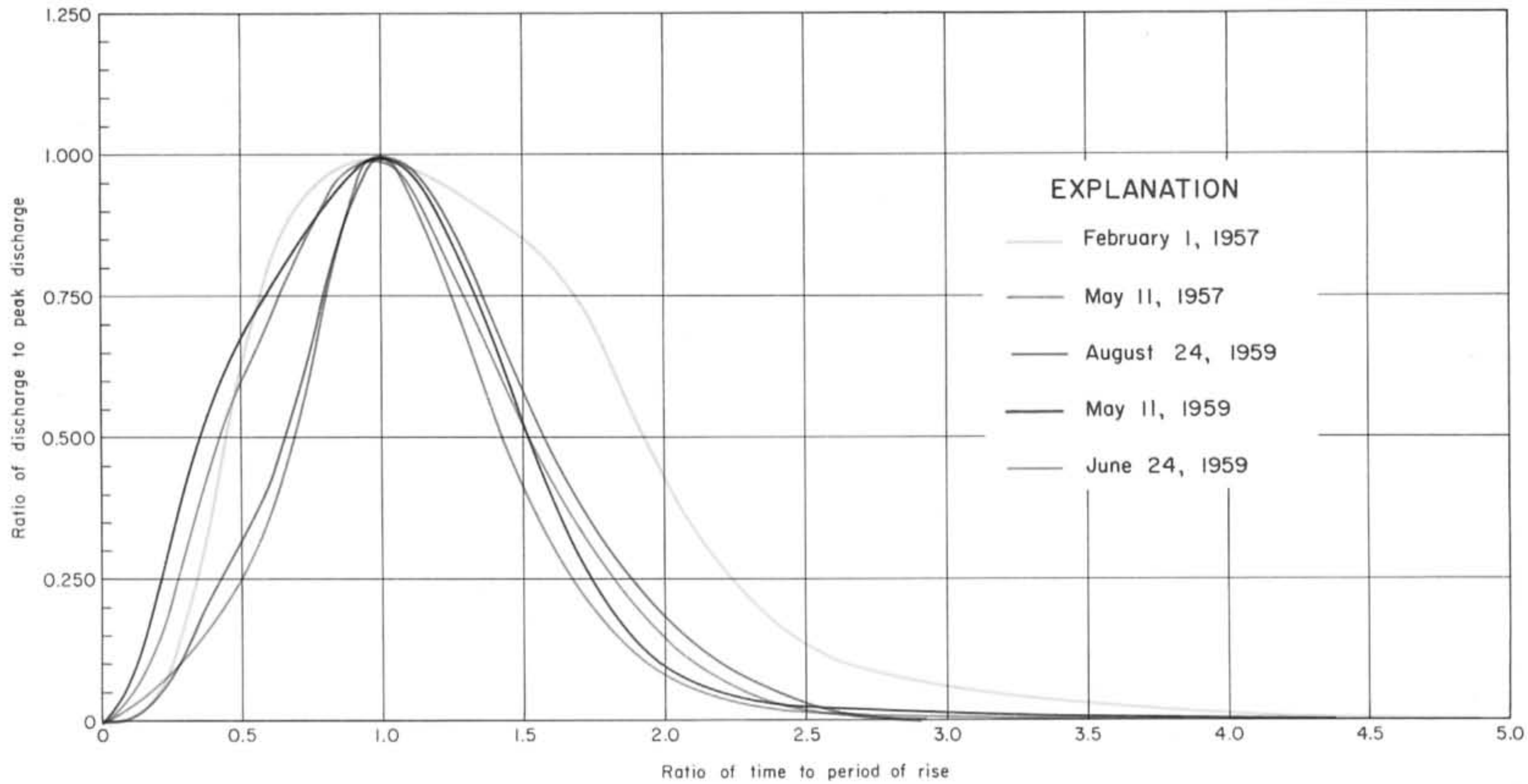

Figure 14

Dimensionless Two-Hour Unit Hydrographs, Pin Oak Creek Watershed

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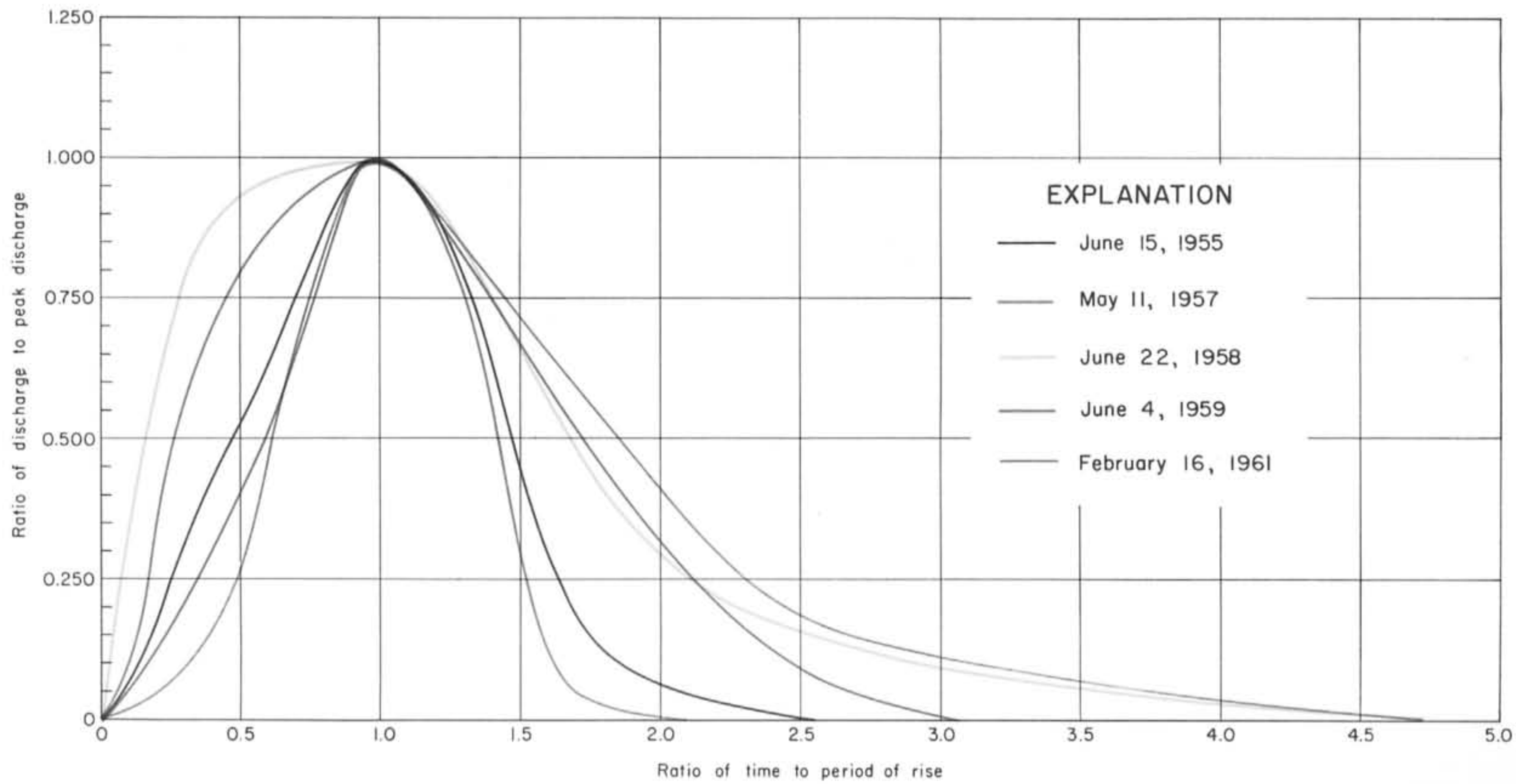

Figure 15

Dimensionless Two-Hour Unit Hydrographs, Mukewater Creek Watershed

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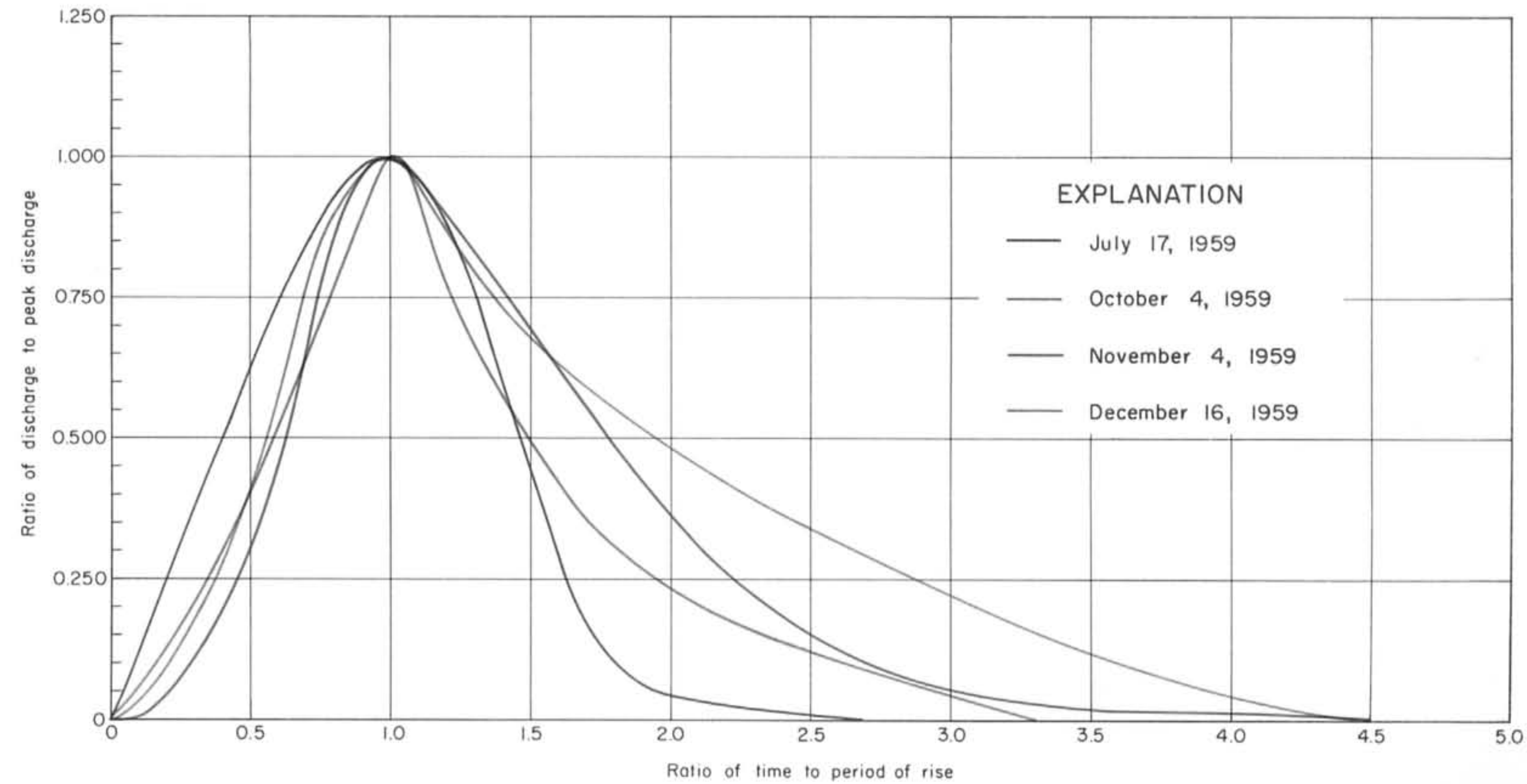

Figure 16
Dimensionless Two-Hour Unit Hydrographs, Little Elm Creek Watershed
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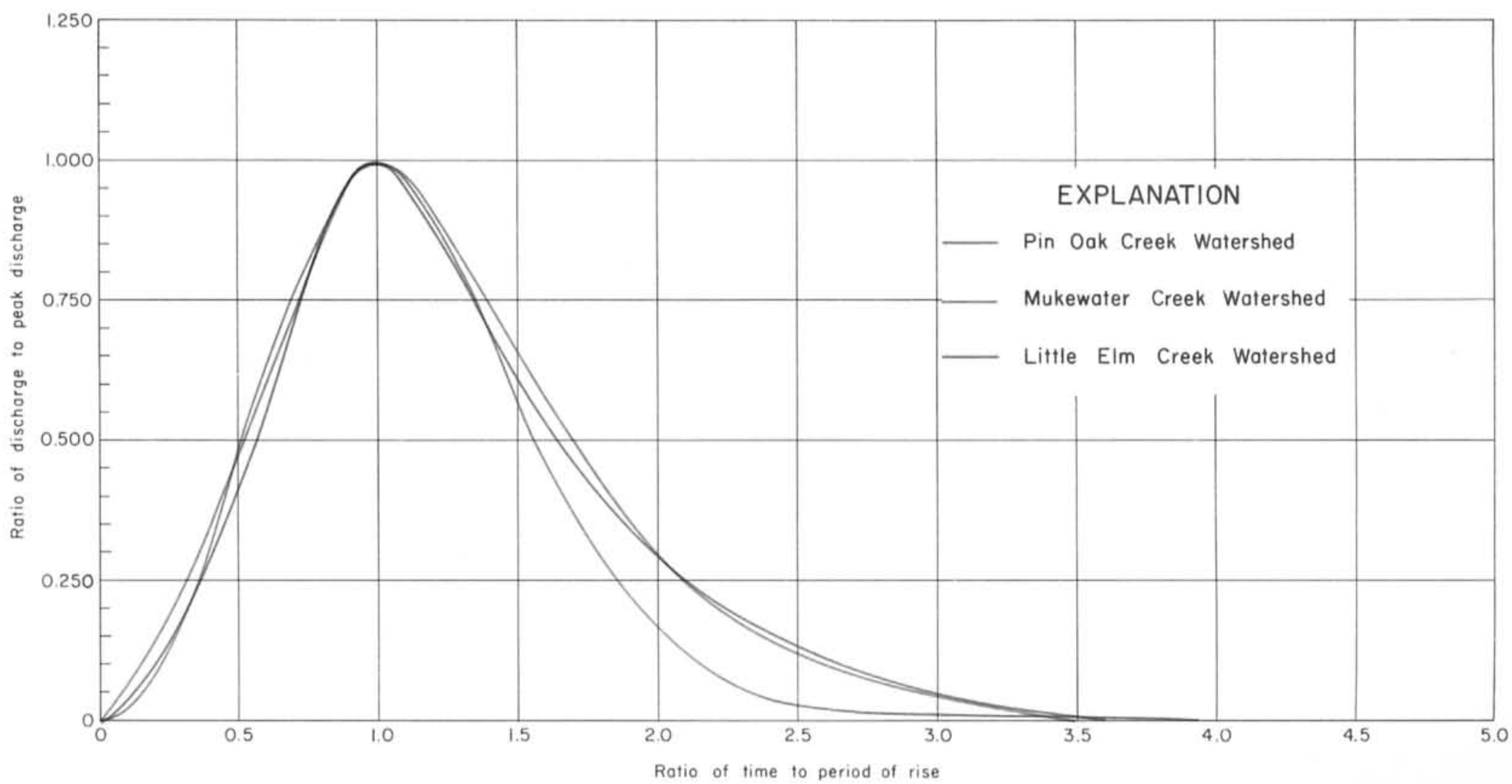

Figure 17

Average Dimensionless Two-Hour Unit Hydrographs for Each Watershed

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Table 4.--Coordinates of average dimensionless two-hour hydrographs

Dimensionless hydrograph abscissas T/T_p	Dimensionless graph ordinates Q/Q_p		
	Little Elm Creek	Pin Oak Creek	Mukewater Creek
0	0	0	0
.05	.019	.007	.033
.10	.042	.021	.069
.15	.071	.045	.108
.20	.105	.080	.150
.30	.192	.169	.241
.40	.296	.318	.345
.50	.412	.484	.462
.60	.550	.635	.580
.70	.711	.768	.715
.80	.871	.871	.840
.85	.924	.916	.895
.90	.967	.953	.948
.95	.994	.991	.988
1.00	1.000	1.000	1.000
1.05	.987	.994	.995
1.10	.951	.975	.981
1.15	.913	.940	.951
1.20	.875	.897	.911
1.30	.793	.800	.837
1.40	.708	.690	.750
1.50	.621	.569	.673
1.60	.538	.462	.590
1.70	.462	.364	.508
1.80	.401	.284	.429
1.90	.348	.219	.359
2.00	.298	.159	.301
2.10	.257	.113	.251
2.20	.220	.082	.211
2.30	.189	.055	.178
2.40	.160	.035	.150
2.50	.137	.027	.126
2.60	.115	.021	.103
2.70	.097	.017	.086
2.80	.080	.013	.070
2.90	.064	.012	.056
3.00	.050	.011	.041
3.10	.037	.0105	.030
3.20	.028	.010	.020
3.30	.019	.008	.012
3.40	.011	.007	.005
3.50	.004	.005	0
3.60	0	.004	--
3.70	--	.003	--
3.80	--	.002	--
3.90	--	0	--

existing empirical hydrographs and as an aid in the determination of average unit hydrograph shapes for each basin.

The average 2-hour unit hydrographs for each study area are shown in Figure 18. They were developed by computing points on the time scale using the dimensionless graph abscissas and the average period of rise for each basin. The discharge ordinates were then computed from the dimensionless graph ordinates such that the area under each graph would equal 1 inch of runoff from each respective watershed. The 2-hour unit graphs indicate the effect of varying basin characteristics on the shape of the unit hydrograph. One can see that the greatest unit graph peak discharge does not occur from the largest drainage area. It should be noted that although the drainage areas of Little Elm Creek and Mukewater Creek are nearly the same, the periods of rise, peak discharges, and time bases are significantly different. The area under the unit graph for Pin Oak Creek is significantly smaller due to the much smaller drainage area.

DISCUSSION OF RESULTS

S-Curve Analyses

The S-curve analysis was performed in order to make a better estimate of the duration of runoff producing rainfall. The task of picking a rainfall excess duration would have been more difficult without the aid of the S-curve analysis. Without having a detailed knowledge of the drainage basin, it was almost impossible to estimate the rainfall excess duration from rainfall records. In many cases, the value of rainfall excess duration estimated on the basis of rainfall records was not the value indicated as best in the S-curve analysis. In almost every one of these cases, the rainfall excess duration was underestimated. When the rainfall records were restudied in light of the

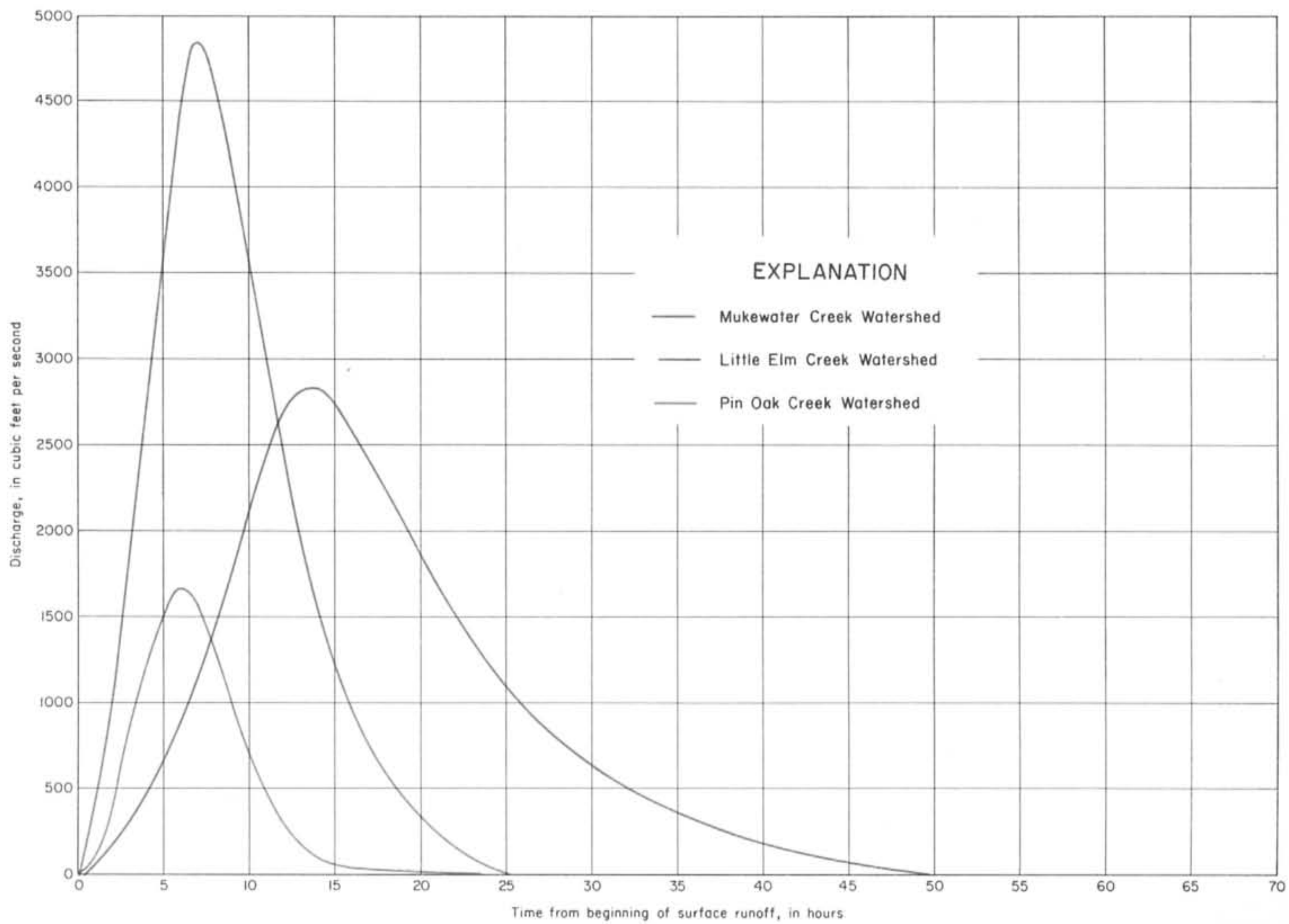

Figure 18

Average Two-Hour Unit Hydrographs for Each Watershed

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results of the S-curve analysis, the revised estimates were considered to be better than the original ones.

The main weakness in the method used for the S-curve analyses was the unavailability of a monotone increasing function for the S-curve. If such an equation had been available to fit to the computed S-curve points, the S-curve error norms would have been more meaningful. In this study, the values of the S-curve error norms could only be used as qualitative measures. In 3 of the 17 S-curve evaluations, the norms were not used to indicate the best duration. The norms indicating the comparison with the equilibrium flow were superior to the S-curve error norms for indicating the best S-curve. Even considering all the norms, it was still necessary to study the computed S-curve ordinates to make the best estimate of the rainfall excess duration. The most accurate way of selecting the best S-curve would be to study the actual plots of trial S-curves. For a great number of trials, however, this procedure becomes prohibitive. It is believed that the computer program used in this study was sufficiently accurate to be used as an aid in the estimate of rainfall excess duration.

This method presents the interesting possibility that if adequate rainfall records are not available and the rainfall excess duration is desired for the hydrograph of surface runoff for an area, the duration could be determined by trial and error utilizing a method similar to the one used in this investigation. It would be helpful to have an estimate of the period of rainfall so as to reduce the number of trials.

Unit Graph Analyses

The purpose of the unit graph analysis was to develop average unit and dimensionless hydrograph shapes for each of the watersheds studied. The basic hydrograph data were used in this portion of the study without adjustment,

except in one case. In this one instance, the rising limb of one of the original was modified to negate the effect of antecedent precipitation.

The derived unit graphs for durations determined in the S-curve analysis were all adjusted to unit graphs for 2-hour durations. The rainfall excess duration of 2 hours was chosen as the common unit graph duration for several reasons. As shown in Table 3, most of the original hydrographs had rainfall excess durations of 2 hours or less. Linsley (1949) stated that the unit graph determined by adjusting a unit graph of long duration to one of shorter duration might exhibit a peak discharge that was lower than the peak of an actual unit graph for the shorter duration. The reason for the higher peak is that abrupt rainfalls sometime cause abrupt translatory waves to be generated. Two hours was considered to be a duration that was short enough to be desirable and yet long enough to cause a minimum amount of adjustment error. It would be possible also to adjust 2-hour unit graphs to hydrographs for durations of multiples of 2 hours, and this was considered to be desirable.

The adjustment process was believed to have effected an improvement in the data. This is evidenced by the data in Table 5, which shows a comparison of the characteristics of unit graphs for Little Elm Creek before and after adjustment. If the unit graph of July 17, 1959 is assumed to be a representative 2-hour graph for the area, it would seem that the adjustment of the December 16, 1959 unit hydrograph did not have a great effect on the accuracy of the peak.

To note the effect of the smoothing operation in the unit graph analysis, known 2-hour storms were run through the computer analysis. As noted in Table 5 for the July 17, 1959 storm, little change was made in the pertinent parameters. Only minor change was noted in the individual unit graph points.

Table 5.--Comparison of unit graph characteristics for Little Elm Creek Watershed before and after adjustment

Storm date	Original unit graph parameters				Two-hour unit graph parameters		
	Duration	Period of rise	Time base	Peak discharge	Period of rise	Time base	Peak discharge
July 17, 1959	2.0	15.0	40.0	3,032	15.0	39.5	3,024
Oct. 4, 1959	3.0	11.0	70.0	2,632	11.0	51.0	2,688
Nov. 4, 1959	3.0	13.5	74.5	2,841	13.0	58.5	2,882
Dec. 16, 1959	7.0	18.5	61.0	2,472	15.5	46.5	2,841

The original data for the hydrographs measured at Pin Oak Creek on February 1, 1957 and at Mukewater Creek on June 22, 1958 were studied to determine the cause for the disagreement between these curves and trends for the watersheds. It is believed that the elongated recession curve on the February 1, 1957 hydrograph was caused by a small amount of rainfall that fell after the initial burst of rainfall producing the major portion of runoff represented by the hydrograph. The sharp rise of the storm of June 22, 1958 at Mukewater Creek seems to have been caused by the intensity of the rainfall. Almost 2 inches of rain fell in about 1-1/2 hours. The rain appeared to have been fairly well distributed over the area. Average intensities of up to 0.42 inches in 10 minutes were noted. It appears that this rainfall caused the hydrograph of runoff to exhibit an unusually sharp period of rise. Detailed data on the rainfall related to the hydrograph of May 11, 1957 at Mukewater Creek were not available for analysis.

At the time the storm causing the unit graph of February 16, 1961 on Mukewater Creek occurred, runoff from approximately 8 percent of the watershed was partially controlled. This unit graph was included in this study because it was considered that the degree of control was not significant enough to necessitate excluding it.

In making the average dimensionless graphs, care was taken not to weigh the unusually shaped hydrographs discussed above as heavily as the curves for which there was better agreement. The average dimensionless hydrographs shown in Figure 17 exhibit several interesting features. The average dimensionless graphs for Little Elm Creek and Mukewater Creek compare closely, while the dimensionless graph for Pin Oak Creek deviates from the other two. Since Little Elm Creek and Mukewater Creek have drainage areas of approximately the same size, size of drainage area is indicated as a significant factor in the shape of the dimensionless hydrograph. Based on these three curves, it seems that for smaller watersheds more hydrograph area should be concentrated about the peak.

It is believed that Figure 18 indicates that size of drainage area is not the most significant item in determining the shape of the unit graph. Drainage area is significant mainly in determining the volume of runoff under the curve. The pertinent parameters of the unit graph such as period of rise, peak discharge, time base of hydrograph, and others are functions of a number of basin characteristics. Without an adequate means of estimating such hydrograph parameters as period of rise and peak discharge based on basin characteristics, the most accurate dimensionless hydrograph shape would be of little use.

Comparison of Results with Empirical Methods

The results of this study were compared with two widely-used empirical hydrograph shapes. The two empirical curves chosen for comparison with the computed dimensionless graphs were those of Commons (1942) and Mockus (Soil Conservation Service, 1957). The methods developed by the Soil Conservation Service (1957) are used frequently in Texas for the design of small watershed projects. These methods were not discussed under the section of this report

concerning previous investigations because they are reported in design manuals with little reference to the methods of development.

The average dimensionless hydrographs were compared with the Commons' hydrograph expressed in dimensionless form in Figure 19 and the Mockus dimensionless hydrograph (Soil Conservation Service, 1957) in Figure 20. The dimensionless hydrograph as developed by Mockus is asymptotic to the abscissa. The curve was terminated at a ratio of time to period of rise of 5.0 since this can be done without inducing much error.

On the basis of these comparisons, both the empirical and the computed curves can be seen to be similar in the rising limbs. There is considerable variation, however, in the base widths of the empirical curves and the curves computed from the small watershed data. There is also some variation between the falling limbs of the actual dimensionless graphs and those of Commons and Mockus. Mockus' dimensionless hydrograph compares more closely to the average dimensionless graphs developed in this study than does Commons' curve.

For a given quantity of runoff both Mockus' and Commons' empirical curves would result in lower peak discharges than would the dimensionless plots developed in this study. The cause for the lowering of the peak would be the greater amount of hydrograph area under the falling limbs of the two empirical curves.

In the SCS (Soil Conservation Service) design manual (1957), an equation was developed for the peak discharge of a triangular unit graph. This equation indicates that there is a linear relationship between the unit graph peak discharge and the ratio of drainage area to period rise. The relationship as developed by the SCS is expressed in equation 15.

$$Q_p = 484 \frac{A}{T_p} \tag{15}$$

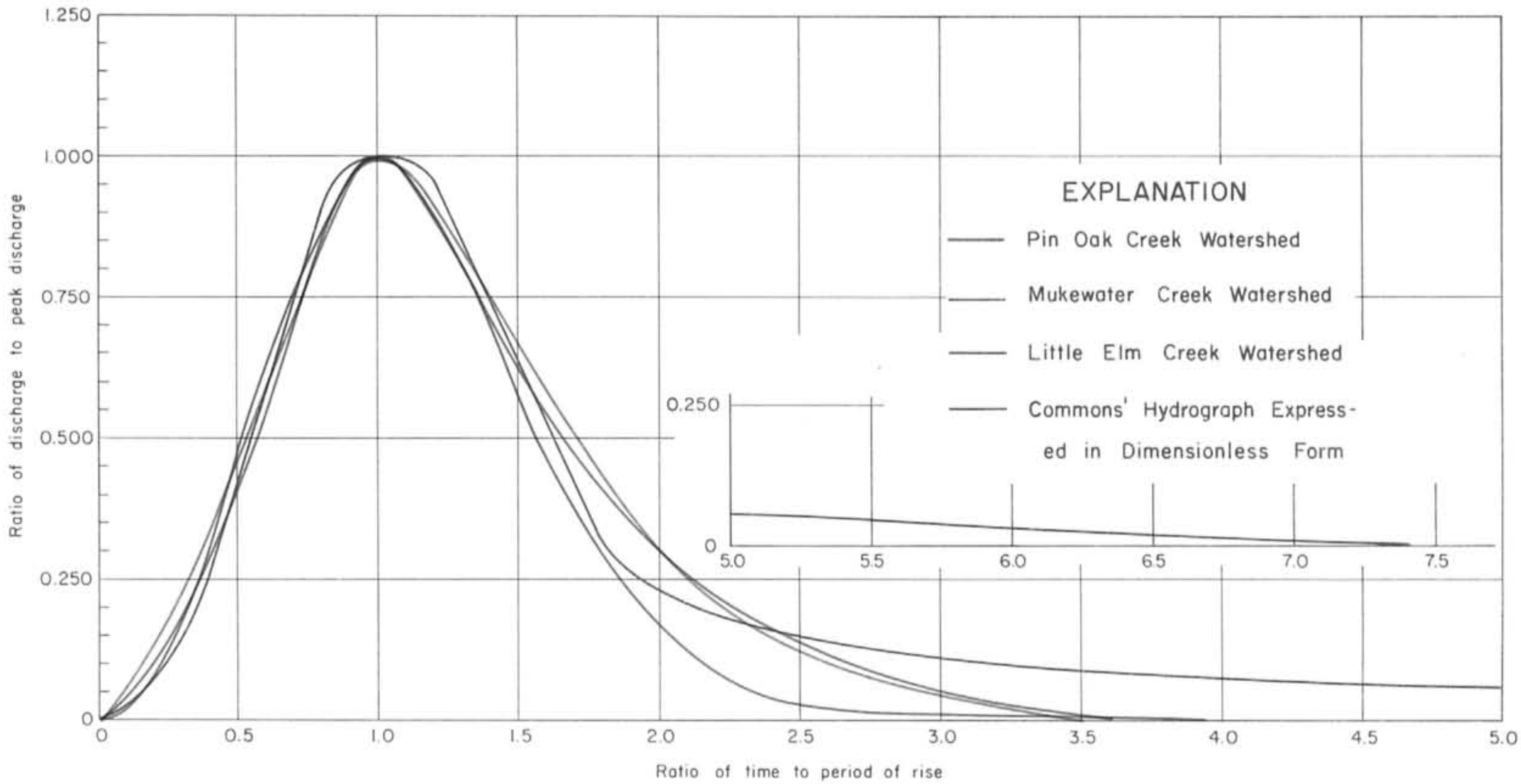

Figure 19

Comparison of Computed Dimensionless Hydrographs with Commons' Hydrograph Expressed in Dimensionless Form

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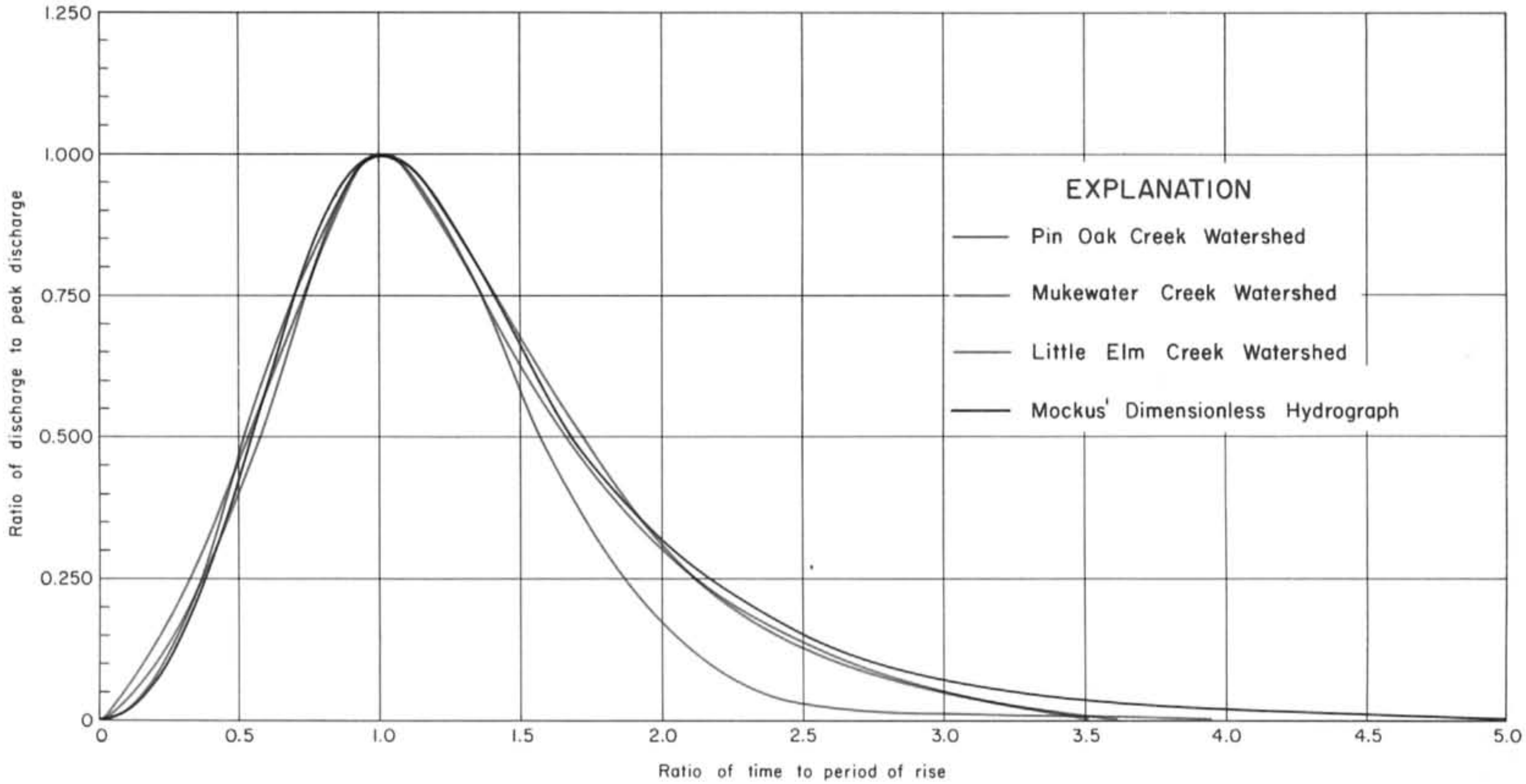

Figure 20

Comparison of Computed Dimensionless Hydrographs with Mockus' Dimensionless Hydrograph

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In this equation Q_p, A, and T_p represent unit graph peak discharge in cfs, drainage area in square miles, and period of rise in hours, respectively.

The 2-hour unit graph peak discharges determined in this study were plotted versus the ratio of drainage area to period of rise. The results of this plot along with a plot of equation 15 are shown in Figure 21. A linear relationship as expressed by equation 16 is indicated.

$$Q_p = 461 \, A/T_p + 340 \tag{16}$$

The relation expressed in equation 16 would result in higher unit graph peak discharges than would the expression in equation 15 up to the point at which the time to period of rise ratio equals 20. At this point the two lines cross and equation 15 provides larger peaks than equation 16. One possible reason for the peak discharges determined in this study to exceed the peak discharges defined by the SCS equation would be the greater amount of dimensionless graph area under the falling limb of the Mockus dimensionless graph.

In considering the possibility of being able to estimate the unit graph period of rise, a linear plot on semilogarithmic paper was found to exist between the period of rise and a dimensionless quantity made up of the length of main stream squared, square root of representative main stream slope, and drainage area. Such a plot indicates that a relation probably exists between the period of rise and a limited number of basin characteristics. Because only three points were available, this plot was not included in this report.

Figure 21 illustrates the importance of the period of rise in the relation between unit graph peak discharge and the ratio of drainage area to period of rise. The center point on either plot in the figure is the point representing the largest drainage area, Little Elm Creek. The characteristic 2-hour unit graph for Little Elm Creek exhibited a lower peak discharge, longer period of rise, and longer time base than the characteristic graph for the comparable

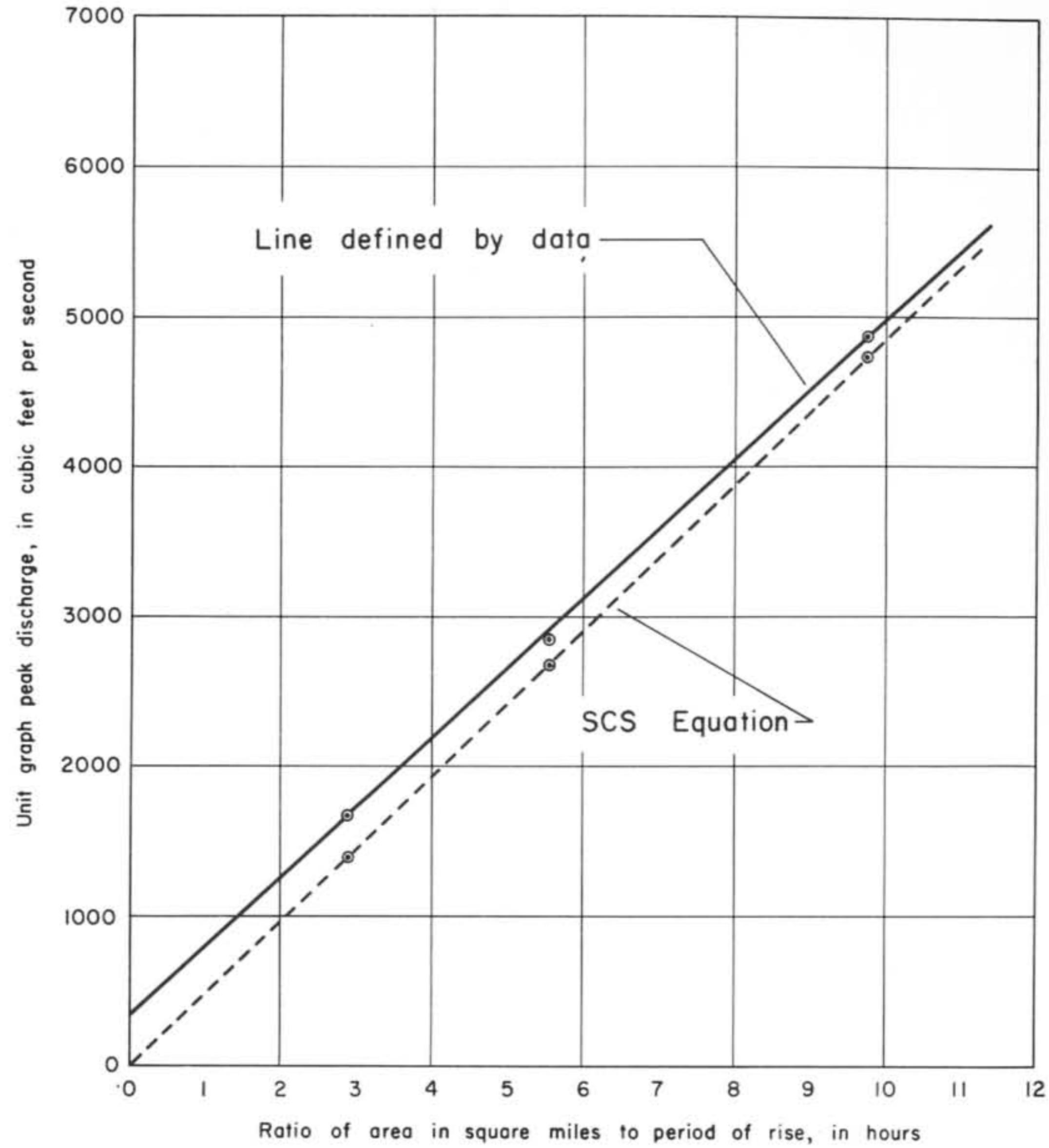

Figure 21

Comparison of Computed Two-Hour Unit Hydrograph Peaks with Those Forecast Using SCS Equation

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sized drainage area, Mukewater Creek. The cause for this difference is probably the difference in basin characteristics. Little Elm Creek drains a long narrow basin with one main stream channel. Mukewater Creek is a shorter fan-shaped basin with two main stream channels draining the upper watershed.

Although there is some disagreement between the dimensionless graphs determined in this study and the curves of Commons and Mockus, it is relatively minor. A study of Figures 19 and 20 indicates that the use of any one of the plotted dimensionless curves to represent the dimensionless unit hydrograph for any one of the watersheds studied would not result in serious error. However, Figure 18 graphically illustrates the importance of having reliable methods for estimating the unit graph peak discharge and period of rise when developing unit hydrographs from dimensionless hydrographs. The need for such methods was indicated by Morgan and Johnson (1962) when they showed that the several synthetic unit graph procedures that they evaluated provided better agreement with actual unit graphs when the observed lag time was used in place of the computed lag time.

CONCLUSIONS

The following conclusions may be drawn from this study:

1. The S-curve can be used in the absence of multiple correlation, infiltration capacity, or other such data to estimate by trial and error the rainfall excess duration.

2. The dimensionless 2-hour unit hydrographs developed in this study indicate that the falling limbs of the Commons and Mockus hydrograph shapes may need revision in order to be applied to watersheds in Texas of less than 100 square miles in area.

3. As a result of the study of the basic data, it is believed that the temporal and spacial distribution of the storm was one of the most important factors influencing the hydrograph shape.

4. The dimensionless and 2-hour unit hydrographs for the Little Elm Creek and Mukewater Creek Watersheds indicate that a reasonably accurate means of estimating the period of rise and peak discharge is necessary when using empirical hydrographs for design purposes.

5. Figures 19 and 20 illustrate that only minor differences occur in the dimensionless graphs. This suggests that an average dimensionless graph and reliable estimates of only two parameters, period of rise and peak discharge, may be sufficient to define the shape of the unit hydrograph.

SUGGESTIONS FOR FUTURE STUDY

While providing needed data and methods of analysis, this study has raised more questions than it has answered. Knowledge concerning the temporal distribution of runoff from floods is certainly lacking. Therefore, there is ample room for further investigation. Some suggested topics that need further research are the following:

1. A monotone increasing function for the S-curve needs to be developed, which can be satisfactorily fitted to S-curve data.

2. More study is needed of probable correlations between basin characteristics and the unit graph properties, period of rise and peak discharge, in Texas.

3. Research needs to be initiated for developing instantaneous unit graphs to determine whether these better represent the effects of basin characteristics on the shape of the hydrograph.

4. More basic data needs to be analyzed from small watersheds to check out the methods used in this study and provide additional unit graph characteristics for correlation with basin characteristics.

5. It would be helpful to perfect a method for fitting unit graph data to equations in order that computers might be used to generate synthetic unit graphs.

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